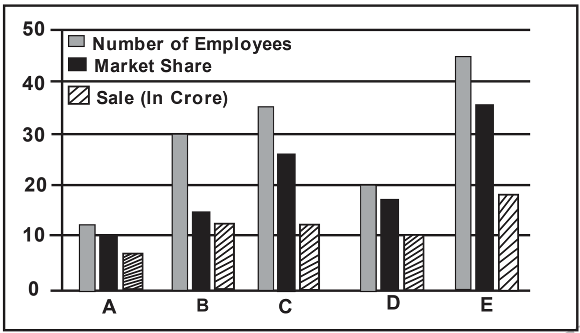
CSIR NET QUESTION PAPER
16-SEPTEMBER-2022

PART - A:

Qus 1.

The graph shows number of employees, market share (as % by number of units sold), and sale (in Rs. crore) for five companies A,B,C,D,E



(a) A has the highest market share per employee, C has the highest sale for its market share.

(b) C has the highest market share per employee, B has the highest sale for its market share.

(c) D has the highest market share per employee, E has the highest sale for its market share.

(d) A has the highest market share per employee, B has the highest sale for its market share.

Qus 2.

A battalion consists of elephants, horses and soldiers totaling to 3500. There are twice as many horses as elephants and one-fourth of the soldiers are riding these animals. In the stand still position, number of feet on ground is 7500. The number of horses in the battalion is.

(a) 525 **(b)** 625

(c) 550 **(d)** 600

Qus 3.

A ranks 10th from both the top and the bottom in merit among the girls in her class. B ranks 6th from the top and 16th from the bottom among boys in the same class. If A is immediately ahead of B in merit order, her rank in the entire class would be.

(a) 16th from the top and 26th from the bottom

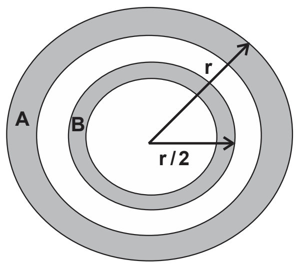
(b) 15th from the top and 26th from the bottom

(c) 15th from the top and 27th from the bottom

(d) 16th from the top and 27th from the bottom

Qus 4.

There are two concentric circular tracks A and B of width 2 m each as shown in the figure. If $r = 30\,\text{m}$, what is the ratio of the areas of Track A to Track B ?

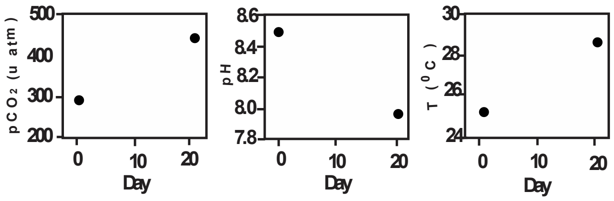


(a) $28/13$ **(b)** $2/1$

(c) $29/14$ **(d)** $5/3$

Qus 5.

Given figure represents pH, partial pressure of $CO_2\,(pCO_2)$, and temperature (T) in an experiment conducted in a water sample over 20 days. Which of the following statements can definitely be made based on this experiment ?



(a) High CO_2 causes global warming

(b) High temperature causes acidification

(c) There is a decrease in pH and an increase in both T and pCO_2 over 20 days

(d) pH and pCO_2 are positively correlated while pH and T are inversely.

Qus 6.

If 90 people are to be seated randomly in 15 rows of 6 seats each, what is the probability that a person gets, a seat at either and of a row ?

(a) $1/2$ **(b)** $1/4$

(c) $1/3$ **(d)** $1/15$

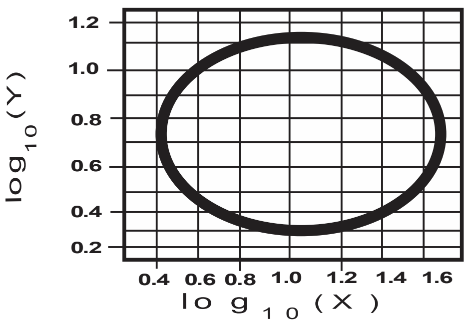
Qus 7.

In a test with multiple choice questions, candidates get 4 marks for correct answer and lose 1 mark for an incorrect answer. Two candidates A and B attempting 18 and 13 questions, respectively, secure equal marks,. How many more INCORRECT answers does A have compared to B ?

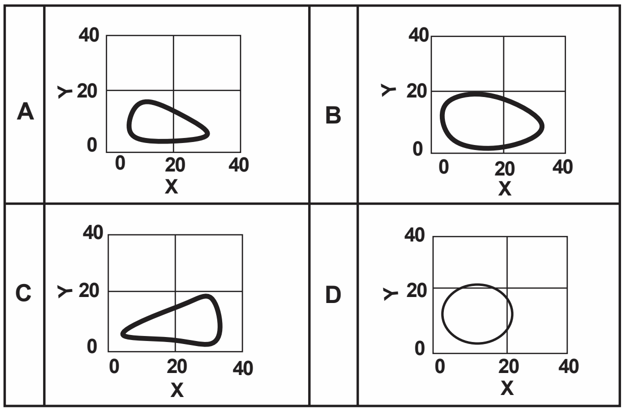
(a) 3 **(b)** 4

(c) 5 **(d)** 6

Qus 8.



Which one of the following drawn on a linear scale, represents the circle shown in the figure above ?



Select the CORRECT option

(a) A **(b)** B

(c) C **(d)** D

Qus 9.

A 5 kg watermelon contains 99% water by weight. Some of the water evaporates and the melon now contains 98% water by weight. What is the weight (in kg) of watermelon now

(a) 4.5 **(b)** 2.5

(c) 4.8 **(d)** 4.9

Qus 10.

Consider the following four statements.

Statement 1: "Statement 3 is true"

Statement 2: "Statement 1 is true"

Statement 3: " Statement 1 is true and statement 2 is false"

Statement 4: "Statement 1,2 and 3 are false

Which of the above statements must be true for the four statements to be mutually consistent ?

(a) Statement 1 **(b)** Statement 2

(c) Statement 3 **(d)** Statement 4

Qus 11

A set of 27 similar looking coins has 26 identical coins and one dummy coin having less weight. What is the minimum number of weighings that will ensure identification of the dummy coin using a two-pan balance ?

(a) 3 **(b)** 4

(c) 5 **(d)** 6

Qus 12.

The wholesale price per unit of an item is C_0 up to first 19 units. The unit price falls by 10% if 20 to 29 units are purchased and by another 10 % if 30 or more units are purchased. If 120 units are bought, the total price paid is approximately.

(a) $99C_0$ (b) $97C_0$

(c) $91C_0$ (d) $81C_0$

Qus 13.

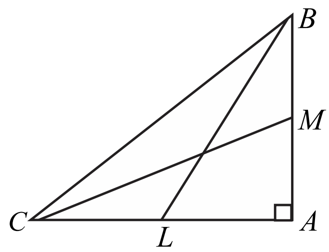
A 360 ml aqueous solution contains 40% alcohol. How much will be the approximate percentage of alcohol if 3600 ml of water is added to the solution ?

(a) 2.6 (b) 3.6

(c) 4.0 (d) 1.0

Qus 14.

Consider a right angled triangle BAC with medians CM and BL having the same length. The ratio of the length of BC to the of ML is



(a) 2 (b) $3/4$

(c) $4/3$ (d) 1

Qus 15.

The arithmetic mean of five numbers is zero. The numbers may not be distinct. Which of the following must be true ?

(a) The product of the numbers is zero
(b) At most two of these numbers are positive
(c) There cannot be exactly one zero
(d) There cannot be exactly one non-zero number

Qus 16.

Of all the English magazines published in a country, magazine M is read by the highest number of readers. It necessarily follows that

(a) M is the most popular english magazine published in the country
(b) M is the most popular english magazine in the country
(c) M is the most popular magazine in the countary
(d) The study has not considered the readership of english magazines published outside the country

Qus 17.

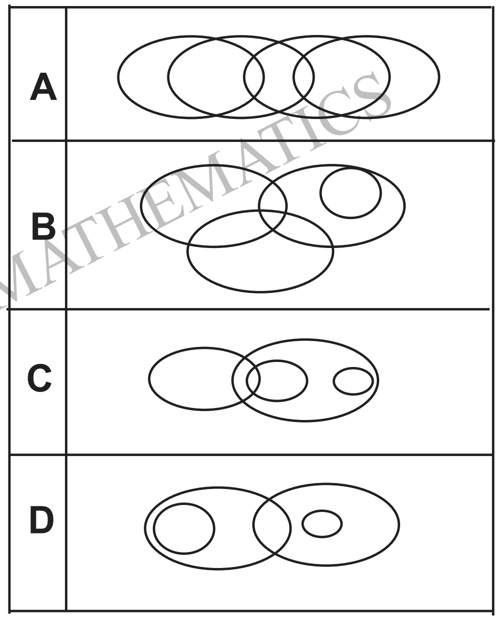
Two digital clocks show times 09h 13m and 09h 17m, respectively, at one instant. Exactly 30 seconds later the clocks show 09h14m and 09h 17m, respectively. Which one of the following options is a possible difference between the times maintained by the two clocks

(a) 3m 00s (b) 30s

(c) 4m 00s (d) 4m 30s

Qus 18.

An appropriate diagram showing the relationship between the categories FOOD, VEGETABLES, ROOTS and ICECREAMS is



Select the correct Option

(a) A (b) B
(c) C (d) D

Qus 19.

The number of three digit PINs, in which the third digit is the sum of the first two digits, is

(a) 55 (b) 9

(c) 45 (d) 11

Qus 20.

On a track of 200 m length, S runs from the starting point and R starts 20 m ahead of S at the same time. Both reach the end of the track at the same times. S runs at a uniform speed of 10m/s. If R also runs at a uniform speed, then how much more time would

R take to run the entire course ?

(a) 0.5 second (b) 1.0 second
(c) 1.5 second (d) 2.2 second

PART - B:

Qus 21.

Let $A=(a_{i,j})$ be a real symmetric 3×3 matrix. Consider the quadratic form $Q(x_1,x_2,x_3)=x^tAx$ where $x=(x_1,x_2,x_3)^t$ Which of the following is true ?

(a) If $Q(x_1,x_2,x_3)$ is positive definite, then $a_{i,j}>0$ for all $i\neq j$,

(b) If $Q(x_1,x_2,x_3)$ is positive definite, then $a_{i,i}>0$ for all i

(c) If $a_{i,i}>0$ for all $i\neq j$, then $Q(x_1,x_2,x_3)$ is positive definite.

(d) If $a_{i,i}>0$ for all i, then $Q(x_1,x_2,x_3)$ is positive definite.

Qus 22.

Suppose $(a_n)_{n\geq1}$ and $(b_n)_{n\geq1}$ are two bounded sequences of real numbers.
Which of the following is true ?

(a) $\limsup_{n\to\infty}\left(a_n+(-1)^n b_n\right)=\limsup_{n\to\infty}a_n+\left|\limsup_{n\to\infty}b_n\right|$

(b) $\limsup_{n\to\infty}\left(a_n+(-1)^n b_n\right)\leq\limsup_{n\to\infty}a_n+\limsup_{n\to\infty}b_n$

(c) $\limsup_{n\to\infty}\left(a_n+(-1)^n b_n\right)\leq\limsup_{n\to\infty}a_n+\left|\limsup_{n\to\infty}b_n\right|+\left|\liminf_{n\to\infty}b_n\right|$

(d) $\limsup_{n\to\infty}\left(a_n+(-1)^n b_n\right)$ may not exist

Qus 23.

Let $a_n=n+n^{-1}$. Which of the following is true for the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{a_{n+1}}{n!}$?

(a) It does not converge.

(b) It converges to $e^{-1}-1$

(c) It converges to e^{-1}

(d) It converges to $e^{-1}+1$

Qus 24.

Let us define a sequence $(a_n)_{n\in\mathbb{N}}$ of real numbers to be a Fibonacci-like sequence if $a_n=a_{n-1}+a_{n-2}$ for $n\geq3$. What is the dimension of the $\mathbb{R}$ vector space of Fibonacci-like sequences ?

(a) 1
(b) 2
(c) Infinite and countable
(d) Infinite and uncountable

Qus 25.

Let $\mathbb{R}$ be the field of real numbers. Let V be the vector space of real polynomials of degree at most 1. Consider the bilinear form $\langle\,,\rangle:V\times V\to\mathbb{R}$ given by $\langle f,g\rangle=\int_0^1 f(x)g(x)\,dx$. Which of the following is true ?

(a) For all nonzero real numbers a,b there exists a real number c such that the vectors $ax+b,x+c\in V$ are orthogonal to each other.

(b) For all nonzero real numbers b, there are infinitely many real numbers c such that the vectors $x+b,x+c\in V$ are orthogonal to each other.

(c) For all positive real numbers c, there exist infinitely many real numbers a,b such that the vectors $ax+b,x+c\in V$ are orthogonal to each other.

(d) For all nonzero real numbers b, there are infinitely many real numbers c such that the vectors $b, x+c \in V$ are orthogonal to each other.

Qus 26.

Consider the series $\sum_{n=3}^{\infty} \frac{a^n}{n^b (\log_e n)^c}$ for which values of $a, b, c \in \mathbb{R}$, does the series Not converges ?

(a) $|a| < 1, b, c \in \mathbb{R}$

(b) $a = 1, b > 1, c \in \mathbb{R}$

(c) $a = 1, 1 \geq b \geq 0,\ c < 1$

(d) $a = -1, b \geq 0,\ c > 0$

Qus 27.

Let X, Y be defined by

$$X = \left\{ (x_n)_{n\geq 1} : \limsup_{n\to\infty} x_n = 1, \text{where } x_n \in \{0,1\} \right\}$$

and $Y = \left\{ (x_n)_{n\geq 1} : \lim_{n\to\infty} x_n \text{ does not exist where } x_n \in \{0,1\} \right\}$

Which of the following is true ?

(a) X, Y are countable

(b) X is countable and Y is uncountable

(c) X is uncountable and Y is countable

(d) X,Y are uncountable

Qus 28.

Let $f_n : [0,1] \to \mathbb{R}$ be given by $f_n(t) = (n+2)(n+1)t^n(1-t)$ for all t in $[0,1]$. Which of the following is true ?

(a) The sequence (f_n) converges uniformly.

(b) The sequence (f_n) converges pointwise but not uniformly.

(c) The sequence (f_n) diverges on $[0,1)$

(d) $\lim_{n\to\infty} \int_0^1 f_n(t)\, dt = \int_0^1 \lim_{n\to\infty} f_n(t)\, dt$

Qus 29.

Suppose A and B are similar real matrices, that is there exists an invertible matrix S such that $A = SBA^{-1}$.

Which of the following need not be true ?

(a) Transpose of A is similar to the transpose of B

(b) The minimal polynomial of A is same as the minimal polynomial of B

(c) trace(A) = trace (B)

(d) The range of A is same as the range of B

Qus 30.

Let A be an invertible 5×5 matrix over a field F. Suppose that characteristic polynomials of A and A^{-1} are the same.

Which of the following is necessarily true ?

(a) $\det(A)^2 = 1$

(b) $\det(A)^5 = 1$

(c) $\text{trace}(A)^2 = 1$

(d) $\text{trace}(A)^5 = 1$

Qus 31.

Suppose A is a real $n \times n$ matrix of rank r. Let V be the vector space of all real $n \times n$ matrices X such that $AX = 0$. What is the dimension of V ?

(a) r **(b)** nr

(c) $n^2 r$ **(d)** $n^2 - nr$

Qus 32.

Let D denote a proper dense subset of a metric space X. Suppose that $f : D \to \mathbb{R}$ is a uniformly continuous function. For $p \in X$, let $B_n(p)$ denote the set $\left\{ x \in D : d(x,p) < \frac{1}{n} \right\}$

Consider $W_p = \bigcap_n \overline{f(B_n(p))}$

Which of the following statements is true ?

(a) W_p may be empty for some p in X

(b) W_p is not empty for every p in X and is contained in $f(D)$

(c) W_p is a singleton for every p

(d) W_p is empty for some p and singleton for some p.

Qus 33.

If $\left|e^{e^z}\right|=1$ for a complex number $z=x+iy, x, y\in\mathbb{R}$, then which of the following true ?

(a) $x=n\pi$ for some integer n.

(b) $y=(2n+1)\frac{\pi}{2}$ for some integer n

(c) $y=n\pi$ for some integer n

(d) $x=(2n+1)\frac{\pi}{2}$ for some integer n

Qus 34.

Let R be a commutative ring with identity. Let S be a multiplicatively closed set such that $0\notin S$. Let I be an ideal which is maximal with respect to the condition that $S\cap I=\phi$.

Which of the following is necessarily true ?

(a) I is a maximal ideal

(b) I is a prime ideal

(c) $I=(1)$

(d) $I=(0)$

Qus 35.

Let R be a ring and N the set of nilpotent elements, i.e.

$N=\{x\in R \mid x^n=0 \text{ for some } n\in\mathbb{N}\}$

Which of the following is true ?

(a) N is an ideal in R

(b) N is never an ideal in R

(c) If R is non-commutative, N is not an ideal

(d) If R is commutative, N is an ideal

Qus 36.

Let $f(z)=(1-z)e^{\left(z+\frac{z^2}{2}\right)}=1+\sum_{n=1}^{\infty}a_n z^n$

Which of the following is false ?

(a) $f'(z)=-z^2e^{\left(z+\frac{z^2}{2}\right)}$

(b) $a_1=a_2$

(c) $a_n\in(-\infty,0]$

(d) $\sum_{n=3}^{\infty}|a_n|<1$

Qus 37.

Let X be a connected metric space with at least two points. Which of the following is necessarily true ?

(a) X has finitely many points

(b) X has countably many points but is not finite

(c) X has uncountably many points

(d) No such X exists

Qus 38.

Let G be a simple group of order 168. How many elements of order 7 does it have ?

(a) 6 **(b)** 7

(c) 48 **(d)** 56

Qus 39.

For a positive integer n, let $f^{(n)}$ denote the n^{th} derivative of f.

Suppose an entire function f satisfies $f^{(2)}+f=0$

Which of the following is correct ?

(a) $\left(f^{(n)}(0)\right)_{n\geq 1}$ is convergent

(b) $\lim_{n\to\infty} f^{(n)}(0)=1$

(c) $\lim_{n\to\infty} f^{(n)}(0)=-1$

(d) $\left(\left|f^{(n)}(0)\right|\right)_{n\geq 1}$ has a convergent subsequence

Qus 40.

Let f be a non-constant entire function such that $|f(z)|=1$ for $|z|=1$

Let U denote the open unit disk around 0. Which of the following is False ?

(a) $f(\mathbb{C}) = \mathbb{C}$

(b) f has at least one zero in U

(c) f has at most finitely many distinct zeros in $\mathbb{C}$

(d) f can have a zero outside U

Qus 41.

Let $G:[0,1]\times[0,1]\to\mathbb{R}$ be defined as

$$G(t,x) = \begin{cases} t(1-x) & \text{if } t \le x \le 1 \\ x(1-t) & \text{if } x \le x \le 1 \end{cases}$$

For a continuous function f on $[0,1]$, define

$$I[f] = \int_0^1\int_0^1 G(t,x) f(t) f(x)\, dt\, dx$$

Which of the following is true ?

(a) $I[f] > 0$ if f is not identically zero

(b) There exists non-zero f such that $I[f] = 0$

(c) There is f such that $I[f] < 0$

(d) $I[\sin(\pi x)] = 1$

Qus 42.

Consider the second order PDE

$au_{xx} + bu_{xy} + au_{yy} = 0$ in $\mathbb{R}^2$ for $a, b \in \mathbb{R}$

Which of the following is true ?

(a) The PDE is hyperbolic for $b \le 2a$

(b) The PDE is parabolic for $b \le 2a$

(c) The PDE is elliptic for $|b| < 2|a|$

(d) The PDE is hyperbolic for $|b| < 2|a|$

Qus 43.

For any two continuous functions $f, g: \mathbb{R} \to \mathbb{R}$, define

$$f * g(t) = \int_0^t f(S) g(t-S)\, dS$$

Which of the following is the value of $f * g(t)$ when $f(t) = \exp(-t)$ and $g(t) = \sin(t)$?

(a) $\frac{1}{2}\left[\exp(-t) + \sin(t) - \cos(t)\right]$

(b) $\frac{1}{2}\left[-\exp(-t) + \sin(t) - \cos(t)\right]$

(c) $\frac{1}{2}\left[\exp(-t) - \sin(t) - \cos(t)\right]$

(d) $\frac{1}{2}\left[\exp(-t) + \sin(t) + \cos(t)\right]$

Qus 44. Assume that a paritcle of mass m is constrained to move on the hyperbola $xy = b$ under gravity g, with b being a non-zero constant; here x is the horizontal direction and y is the vertical direction.

Which of the following is Lagrange's equation of motion ?

(a) $m\ddot{x}\left(1 + \frac{b^2}{x^4}\right) - 2\frac{b^2 m}{x^5}\dot{x}^2 - \frac{mgb}{x^2} = 0$

(b) $m\ddot{x}\left(1 + \frac{b^2}{x^3}\right) - 2\frac{b^2 m}{x^5}\dot{x}^2 - \frac{mgb}{x^2} = 0$

(c) $m\ddot{x}\left(1 + \frac{b^2}{x^4}\right) - 2\frac{b^2 m}{x^2}\dot{x}^2 - \frac{mgb}{x^2} = 0$

(d) $m\ddot{x}\left(1 + \frac{b^2}{x^5}\right) - 2\frac{b^2 m}{x^3}\dot{x}^2 - \frac{mgb}{x^2} = 0$

Qus 45.

Let $f: \mathbb{R}^2 \to \mathbb{R}$ be continuous and

$f(t,x) < 0$ if $tx > 0$

$f(t,x) > 0$ if $tx < 0$

Consider the problem of solving the following

$\dot{x} = f(t,x),\ x(0) = 0$

Which of the following is true ?

(a) There exists a unique local solution

(b) There exists a local solution but may not be unique

(c) There may not exist any solution

(d) If local solution exists then it is unique

Qus 46.

Let $u(x,t)$ be a smooth solution to the wave equation

$$(*) \qquad \frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0 \text{ for } (x,t) \in \mathbb{R}^2$$

Which of the following is False ?

(a) $u(x-\theta,t)$ also solves the wave equation $(*)$ for any fixed $\theta \in \mathbb{R}$

(b) $\frac{\partial u}{\partial x}$ also solves the wave equation $(*)$

(c) $u(3x,9t)$ also solves the wave equation $(*)$

(d) $u(3x,3t)$ also solves the wave equation $(*)$

Qus 47.

Let A be the following invertible matrix with real positive entries:

$$A = \begin{pmatrix} 1 & 2 \\ 8 & 9 \end{pmatrix}$$

Let G be the associated Gauss-Seidel iteration matrix. What are the two eigenvalues of G ?

(a) 0 and $4/3$

(b) 0 and $-4/3$

(c) 0 and $16/9$

(d) $4/3$ and $-4/3$

Qus 48.

What is the extremal of the functional

$$J[y] = \int_{-1}^{0} \left(12\,x\,y - (y')^2\right) dx$$

subject to $y(0) = 0$ and $y(-1) = 1$?

(a) $y = x^2$

(b) $y = \frac{2x^2 + x^4}{3}$

(c) $y = -x^3$

(d) $y = \frac{x^2 + x^4}{2}$

Qus 49.

If the incidence matrix of a design is $N = aJ_{tb}$, where a is a positive constant and J_{tb} is a $t \times b$ matrix with every element equal to 1, then the design is

(a) Connected but not orthogonal

(b) Orthogonal but not connected

(c) Neither connected nor orthogonal

(d) Both connected and orthogonal

Qus 50.

Let $X_1, \ldots, X_n$ be a random sample from $N(\theta,\theta)$ distribution, where $N(\theta,\theta)$ denotes a normal distribution with mean θ and variance θ; where θ satisfies $0 < \theta < \infty$ and is unknown. Then, the maximum likelihood estimate of θ.

(a) Is $\dfrac{-1-\left(1+\frac{4}{n}\sum_{i=1}^{n} x_i^2\right)^{\frac{1}{2}}}{2}$

(b) Is $\dfrac{-1+\left(1+\frac{4}{n}\sum_{i=1}^{n} x_i^2\right)^{\frac{1}{2}}}{2}$

(c) Does not exist

(d) Is $\max\left(\frac{\sum_{i=1}^{n} x_i}{n}, 0\right)$

Qus 51.

Let $X_1, \ldots, X_n$ be a random sample from a discrete distribution with probability mass function

$$P(X_1 = 1) = \frac{2(1-\theta)}{2-\theta},\ P(X_1 = 2) = \frac{\theta}{2-\theta}$$

where $\theta \in (0,1)$ is unknown. Let $\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$, Then the method of moments estimator of θ is

(a) $\bar{X}$

(b) $2(1-\bar{X})$

(c) $2(1-\bar{X}^{-1})$

(d) $2(2-\bar{X})^{-1}$

Qus 52.

For a given bivariate data $(y_i, x_i), i=1,...,n$, Analyst A fits Y on X, i.e. $\widehat{Y_i} = \widehat{\alpha_0} + \widehat{\alpha_1} x_i$, while Analyst B fits X on Y, i.e. $\widehat{X_i} = \widehat{\beta_0} + \widehat{\beta_1} y_i$, using the ordinary least squares estimation method. Which of the following pairs is a possible value for $(\widehat{\alpha_1}, \widehat{\beta_1})$?

(a) $(-0.5,\ 2.5)$ **(b)** $(0.5,\ 2.5)$

(c) $(-2.0,\ 0.4)$ **(d)** $(-2.0,\ -0.4)$

Qus 53.

Suppose that X is a random variable such that $P(X \in \{0,1,2\}) = 1$. If for some constant c, $P(X=i) = cP(X=i-1)$, $i=1,2$, then $E[X]$ is

(a) $\dfrac{1}{1+c+c^2}$ **(b)** $\dfrac{c+2c^2}{1+c+c^2}$

(c) $\dfrac{c+c^2}{1+2c}$ **(d)** $\dfrac{3c}{1+2c}$

Qus 54.

Let X and Y be independent random variables with $X \sim$ Uniform $[0, \theta+3]$ $Y \sim$ Uniform $[-\theta-5, 0]$, where $\theta \geq -3$. Then the maximum likelihood estimator of θ based on (X, Y) is

(a) $(5+Y,\ X-3)$

(b) $(-5-Y,\ X-3)$

(c) $(5+Y,\ X-3)$

(d) $(-5-Y,\ X-3)$

Qus 55.

Consider the following maximization problem

maximize $x_1 + 3x_2$

subject to the constraints $x_1 \geq 0,\ x_2 \geq 0$ and

$$x_1 + x_2 \leq 6$$
$$2x_1 + x_2 \leq 8$$
$$x_1 + 2x_2 \leq 9$$

Which of the following is the dual problem ?

(a) Minimize: $6y_1 + 8y_2 + 9y_3$ subject to $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$
$y_1 + 2y_2 + y_3 \geq 1,\ y_1 + y_2 + 2y_3 \geq 3,$

(b) Minimize: $6y_1 + 8y_2 + 9y_3$ subject to $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$
$y_1 + 2y_2 + y_3 \geq 3,\ y_1 + y_2 + 2y_3 \geq 1$

(c) Minimize: $6y_1 + 8y_2 + 9y_3$ subject to $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$
$y_1 + 2y_2 + y_3 \leq 1,\ y_1 + y_2 + 2y_3 \leq 3$

(d) Minimize: $9y_1 + 8y_2 + 6y_3$ subject to $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$
$y_1 + 2y_2 + y_3 \geq 1,\ y_1 + y_2 + 2y_3 \geq 3$

Qus 56.

Let $X_1, X_2, ..., X_n$ be independent and identically distributed random variables with probability density function

$$f(x) = \begin{cases} e^{-(x-\theta)}, & \text{if } x > \theta \\ 0, & \text{otherwise} \end{cases}$$

If $\left(X_{(1)} - \frac{1}{n}\log_e 10, X_{(1)}\right)$ is a $100\beta\%$ confidence interval of θ where $X_{(1)} = \min\{X_i : 1 \leq i \leq n\}$, then the value of β is.

(a) 0.95 **(b)** 0.9

(c) 0.975 **(d)** 0.92

Qus 57.

Let $Y_1, Y_2, ..., Y_{10}$ be independent and identically distributed bivariate normal $N_2(0, \Sigma)$ where $\Sigma = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$. Let $a = (2\ 1)^T$ and

$A=\sum_{i=1}^{10} Y_i Y_i^T$ then the distribution of $\frac{5}{3}\left(a^T A^{-1} a\right)^{-1}$ is

(a) Central chi-square with 8 degrees of freedom
(b) Not chi-square
(c) Central chi-square with 9 degrees of freedom
(d) Non-central chi-square

Qus 58.

Suppose $X\sim$ Binomial $\left(10,\frac{1}{2}\right)$, $Y\sim$ Binomial $\left(11,\frac{1}{2}\right)$, where X and Y are independent. Then, $P(X<Y)$ is

(a) Less than $\frac{1}{2}$

(b) Equal to $\frac{1}{2}$

(c) Greater than $\frac{1}{2}$ but less than or equal to $\frac{10}{11}$

(d) Greater than $\frac{10}{11}$

Qus 59.

In a random experiment a fair coin is tossed once. Then, an unbiased six faced die is rolled N times, where

$$N=\begin{cases}100 \text{ if} & \text{Head appears}\\ 101 \text{ if} & \text{Tail appears}\end{cases}$$

Let Y denote the total number of times 6 appears out of N. Then $P(\text{Head} \mid Y=15)$ equals

(a) $\frac{516}{1021}$ **(b)** $\frac{505}{1021}$

(c) $\frac{201}{1021}$ **(d)** $\frac{1000}{1021}$

Qus 60.

Let X be a random variable with probability density function $f(\bullet)$. Then based on a single observation X, the most powerful test of size 0.2 for testing.

$$H_0: f(x)=\begin{cases}4x^3, & \text{if } 0<x<1\\ 0, & \text{otherwise}\end{cases}$$ against

$$H_1: f(x)=\begin{cases}8x^7, & \text{if } 0<x<1\\ 0, & \text{otherwise}\end{cases}$$, has power

(a) 0.81 **(b)** 0.89

(c) 0.64 **(d)** 0.36

Qus 61.

Which of the given sequences (a_n) satisfy the following identity ?

$\lim_{n\to\infty}\sup a_n=-\lim_{n\to\infty}\inf a_n$

(a) $a_n=1/n$ for all n

(b) $a_n=(-1)^n(1+1/n)$ for all n

(c) $a_n=1+\frac{(-1)^n}{n}$ for all n

(d) (a_n) is an enumeration of all rational numbers in $(-1,1)$

Qus 62.

Consider the function $f:\mathbb{R}^2\to\mathbb{R}$ defined by

$$f(x,y)=\begin{cases}(x-y)^2\sin\frac{1}{x-y} & \text{if } x\neq y\\ 0 & \text{if } x=y\end{cases}$$

Which of the following statements are true ?

(a) f is continuous at $(0,0)$

(b) The partial derivative f_x does not exist at $(0,0)$

(c) The partial derivative f_x is continuous at $(0,0)$

(d) f is differentiable at $(0,0)$

Qus 63.

Consider the following assertions:-

S1: $e^{\cos(t)}\neq e^{2022\sin(t)}$ for all $t\in(0,\pi)$

S2: For each $x > 0$, there exists a $t \in (0, x)$ such that $x = \log_e(1 + x e^t)$

S3: $e^{|\sin(x)|} \le e^{|x|}$ for all $x \in (-1, 1)$

Which of the above assertions are correct ?

(a) Only S1
(b) Only S3
(c) Only S1 and S2
(d) Only S2 and S3

Qus 64.

Let $[x]$ denote the integer part of x for any real number x. Which of the following sets have non-zero lebesgue measure ?

(a) $\left\{x \in [1, \infty) : \lim_{n\to\infty} [x]^n \text{ exists}\right\}$

(b) $\left\{x \in [1, \infty) : \lim_{n\to\infty} \left[x^n\right] \text{exists}\right\}$

(c) $\left\{x \in [1, \infty) : \lim_{n\to\infty} n[x]^n \text{ exists}\right\}$

(d) $\left\{x \in [1, \infty) : \lim_{n\to\infty} [1 - x]^n \text{ exists}\right\}$

Qus 65.

Let $\Omega = \bigcup_{i=1}^{5} (i, i+1) \subset \mathbb{R}$ and $f : \Omega \to \mathbb{R}$ be a differentiable function such that $f'(x) = 0$ for all $x \in \Omega$ and let $g : \mathbb{R} \to \mathbb{R}$ be any function.

Which of the following statements are true ?

(a) If g is continuous, then $(g \circ f)(\Omega)$ is compact set in $\mathbb{R}$

(b) If g is differentiable and $g'(x) > 0$ for all $x \in \mathbb{R}$, then $(g \circ f)(\Omega)$ has precisely 5 elements

(c) If g is continuous and surjective, then $(g \circ f)(\Omega) \cap \mathbb{Q} \ne \varnothing$

(d) If g is differentiable, then $\left\{e^x : x \in (g \circ f)(\Omega)\right\}$ does not contain any non-empty open interval

Qus 66.

Let W be the space of $\mathbb{C}$- linear combinations of the following functions

$f_1(z) = \sin z$, $\quad f_2(z) = \cos z$

$f_3(z) = \sin(2z)$, $\quad f_4(z) = \cos(2z)$

Let T be the linear operator on W given by complex differentiation.

Which of the following statements are true ?

(a) Dimension of W is 3

(b) The span of f_1 and f_2 is a Jordan block of T

(c) T has two Jordan blocks

(d) T has four Jordan blocks

Qus 67.

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$f(x, y) = x^{\frac{1}{3}} y^{\frac{1}{3}} \,(x, y \in \mathbb{R})$

Which of the following statements are true ?

(a) The directional derivative of f exists at $(0,0)$ in some direction

(b) The partial derivative f_x does not exist at $(0,0)$

(c) f is continuous at $(0,0)$

(d) f is not differentiable at $(0,0)$

Qus 68.

For a positive integer $n \ge 2$, let A be an $n \times n$ matrix with entries in $\mathbb{R}$ such that A^{n^2} has rank zero. Let 0_n denote the $n \times n$ matrix with all entries equal to 0.

Which of the following statements are equivalent to the statement that A has n linearly independent eigenvectors ?

(a) $A^n = 0_n$ (b) $A^{n^2} = 0_n$

(c) $A = 0_n$ (d) $A^2 = 0_n$

Qus 69.

For $\alpha > 0$, define $a_n = \dfrac{1 + 2^\alpha + \ldots + n^\alpha}{n^{\alpha+1}}$

What is the value of $\lim_{n\to\infty} a_n$?

(a) The limit does not exist.

(b) $\dfrac{1}{\alpha^2+1}$

(c) $\dfrac{1}{\alpha+1}$

(d) $\dfrac{1}{\alpha^2+\alpha+1}$

Qus 70.

Let P_n be the vector space of real polynomials with degree at most n.

Let $\langle\, , \rangle$ be an inner product on P_n with respect to which $\left\{1, x, \frac{1}{2!}x^2, \ldots, \frac{1}{n!}x^n\right\}$ is an orthonormal basis of P_n. Let $f = \sum_i \alpha_i x^i, g = \sum_i \beta_i x^i \in P_n$. Which of the following statements are true ?

(a) $\langle f, g\rangle = \sum_i (i!)\alpha_i \beta_i$ defines one such inner product, but there is another such inner product.

(b) $\langle f, g\rangle = \sum_i (i!)\alpha_i \beta_i$

(c) $\langle f, g\rangle = \sum_i (i!)^2 \alpha_i \beta_i$ defines one such inner product, but there is another such inner product.

(d) $\langle f, g\rangle = \sum_i (i!)^2 \alpha_i \beta_i$

Qus 71.

Let V be the vector space of polynomials $f(X,Y) \in \mathbb{R}[X,Y]$ with (total) degree less than 3. Let $T: V \to V$ be the linear transfor
The rank of T is at least 3.

mation given by $\dfrac{\partial}{\partial X}$.

Which of the following statements are true ?

(a) The nullity of T is at least 3.

(b) The rank of T is at least 4.

(c) The rank of T is at least 3.

(d) T is invertible.

Qus 72.

Let (X,d) be a finite non-singleton metric space. Which of the following statements are true ?

(a) There exists $A \subseteq X$ such that A is not open in X

(b) X is compact.

(c) X is not connected

(d) There exists a function $f: X \to \mathbb{R}$ such that f is not continuous.

Qus 73.

What is the largest positive real number δ such that whenever $|x-y| < \delta$, we have $|\cos x - \cos y| < \sqrt{2}$?

(a) $\sqrt{2}$ **(b)** $3/2$

(c) $\pi/2$ **(d)** 2

Qus 74.

Let A be an $n \times n$ matrix with entries in $\mathbb{R}$ such that A and A^2 are of the same rank. Consider the linear transformation $T: \mathbb{R}^n \to \mathbb{R}^n$ defined by $T(v) = Av$ for all $v \in \mathbb{R}^n$. Which of the following statements are true ?

(a) The kernels of T and $T \circ T$ are the same.

(b) The kernels of T and $T \circ T$ are of equal dimension.

(c) A must be invertible.

(d) $I_n + A$ must be invertible, where I_n denotes the $n \times n$ identity matrix.

Qus 75.

On the complex vector space $\mathbb{C}^{100}$ with standard basis $\{e_1, e_2, \ldots, e_{100}\}$ consider the bilinear form $B(x,y) = \sum_i x_i y_i$ where x_i and y_i are the coefficients of e_i in x and y respectively. Which of the following statements are true ?

(a) B is nondegenerate.
(b) Restriction of B to all nonzero subspaces is nondegenerate.
(c) There is a 51 dimensional subspace W of $\mathbb{C}^{100}$ such that the restriction $B : W \times W \to \mathbb{C}$ is the zero map.
(d) There is a 49 dimensional subspace W of $\mathbb{C}^{100}$ such that the restriction $B : W \times W \to \mathbb{C}$ is the zero map.

Qus 76.

Let U and V be the subspaces of $\mathbb{R}^3$ defined by

$$U = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid 2x + 3y + 4z = 0 \right\}$$

$$V = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid x + 2y + 5z = 0 \right\}$$

Which of the following statements are true ?
(a) There exists an invertible linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(U) = V$.
(b) There does not exist any invertible linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(V) = U$
(c) There exists a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(U) \cap V \neq \{0\}$ and the characteristic polynomial of T is not the product of linear polynomials with real coefficients.
(d) There exists a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(U) = V$ and the characteristic polynomial of T vanishes at 1.

Qus 77.

For positive integer $n \geq 2$, let $M_n(\mathbb{R})$ denote the vector space of $n \times n$ matrices with entries in $\mathbb{R}$. Which of the following statements are true ?
(a) The vector space $M_n(\mathbb{R})$ can be expressed as the union of a finite collection of its proper subspaces.
(b) Let A be an element of $M_n(\mathbb{R})$. Then, for any real number x and $\varepsilon > 0$, there exists areal number $y \in (x - \varepsilon, x + \varepsilon)$ such that $\det(yI + A) \neq 0$.
(c) Suppose A and B are two elements of $M_n(\mathbb{R})$ such that their characteristic polynomials are equal. If $A = C^2$ for some $C \in M_n(\mathbb{R})$ then $B = D^2$ for some $D \in M_n(\mathbb{R})$.
(d) For any subspace W of $M_n(\mathbb{R})$, there exists a linear transformation $T : M_n(\mathbb{R}) \to M_n(\mathbb{R})$ with W as its image.

Qus 78.

Let $a, b \in \mathbb{R}$ such that $a < b$, and let $f : (a, b) \to \mathbb{R}$ be a continuous function. Which of the following statements are true ?
(a) If f is uniformly continuous then there exist $\alpha \geq 0$ and $\beta \geq 0$ satisfying $|f(x) - f(y)| \leq \alpha |x - y| + \beta$, for all x, y in (a, b)
(b) For every c, d such that $[c, d] \subseteq (a, b)$, if f restricted to $[c, d]$ is uniformly continuous then f is uniformly continuous.
(c) If f is strictly increasing and bounded then f is uniformly continuous.
(d) If f is uniformly continuous then it maps Cauchy sequences into convergent sequences.

Qus 79.

Let a, b be positive integers with $a > b$ and $a + b = 24$. Suppose that the following congruences have a common integer solution

$2x \equiv 3a \pmod 5$, $x \equiv 4b \pmod 5$

Which of the following statements are true ?

(a) $10 \le a - b \le 20$

(b) $3b > a > 2b$

(c) $a > 3b$

(d) $a - b$ is divisible by 5

Qus 80.

For a bounded open connected subset Ω of $\mathbb{C}$, let $f : \Omega \to \mathbb{C}$ be holomorphic. Let (z_k) be a sequence of distinct complex number in Ω converging to z_0. If $f(z_k) = 0$ for all $k \ge 1$ then which of the following statements are necessarily true ?

(a) If f is non-zero, then $z_0 \in \partial\Omega$

(b) There exists $r > 0$ such that $f(z) = 0$ for every $z \in \Omega$ satisfying $|z - z_0| \le r$

(c) If $z_0 \in \Omega$, there exists $r > 0$ such that $f(z) = 0$ on $|z - z_0| = r$.

(d) $z_0 \in \partial\Omega$

Qus 81.

Let

$A = \mathbb{Z}[X]/(X^2 + X + 1, X^3 + 2X^2 + 2X + 6)$

Which of the following statements are true ?

(a) A is an integral domain

(b) A is a finite ring

(c) A is a field

(d) A is a product of two rings

Qus 82.

For an open subset Ω of $\mathbb{C}$ such that $0 \in \Omega$ which of the following statements are true ?

(a) $\{e^z : z \in \Omega\}$ is an open subset of $\mathbb{C}$

(b) $\{|e^z| : z \in \Omega\}$ is an open subset of $\mathbb{R}$

(c) $\{\sin z : z \in \Omega\}$ is an open subset of $\mathbb{C}$

(d) $\{|\sin z| : z \in \Omega\}$ is an open subset of $\mathbb{R}$

Qus 83.

Consider the function $f(n) = n^5 - 2n^3 + n$, where n is a positive integer.

Which of the following statements are true ?

(a) For every positive integer k, there exists a positive integer n such that $f(n)$ is divisible by 2^k

(b) $f(n)$ is even for every integer $n \ge 20$

(c) For every integer $n \ge 20$, either $f(n)$ is odd or $f(n)$ is divisible by 4

(d) For every odd integer $n \ge 21$, $f(n)$ is divisible by 64

Qus 84.

Which of the following statements are necessarily true regarding a group G or order 2022

(a) Let g be an element of odd order in G and S_g the permutation of G given by $S_g(x) = gx$ for $x \in G$. Then S_g is an even permutation.

(b) The set $H = \{g \in G \mid \text{order}(g) \text{ is odd}\}$ is normal subgroup of G.

(c) G has a normal subgroup of index 337

(d) G has only 2 normal subgroups

Qus 85.

Let U be a bounded open set of $\mathbb{C}$ containing 0. Let $f : U \to U$ be holomorphic with $f(0) = 0$. For $n \in \mathbb{N}$, let f^n denote the composition of f done n times, that is

$$f^n = \underbrace{f \circ \ldots \circ f}_{n \text{ times}}$$

While f' denotes the derivative of f.

Which of the following statements are true ?

(a) $(f^n)'(0) = (f'(0))^n$

(b) $f^n(U) \subset U$

(c) The sequence $\left((f'(0))^n\right)_n$ is bounded

(d) $|f'(0)| \le 1$

Qus 86.

Let f be an entire function such that $f(z)^2 + f'(z)^2 = 1$. Consider the following sets $X = \{z : f'(z) = 0\}$.

$Y = \{z : f''(z) + f(z) = 0\}$

Which of the following statements are true ?

(a) Either X or Y has limit point.

(b) If Y has a limit point, then f' is constant.

(c) If X has a limit point, then f is constant.

(d) $f(z) \in \{1, -1\}$ for all $z \in \mathbb{C}$

Qus 87.

Le t p be a prime number and let $\overline{\mathbb{F}_p}$ denote an algebraic closure of the field $\mathbb{F}_p$. We define.

$\delta = \{F \subseteq \overline{\mathbb{F}_p} \mid [F : \mathbb{F}_p] < \infty\}$

Which of the following statements are true ?

(a) δ is an uncountable set

(b) δ is a countable set

(c) For every positive integer $n > 1$, there exists a unique field $F \in \delta$ such that $[F : \mathbb{F}_p] = n$.

(d) Given any two fields $F_1, F_2 \in \delta$, either $F_1 \subseteq F_2$ or $F_2 \subseteq F_1$.

Qus 88.

Which of the following are class equations for a finite group ?

(a) $1+3+3+3+3+13+13=39$

(b) $1+1+2+2+2+2+2+2=14$

(c) $1+3+3+7+7=21$

(d) $1+1+1+2+5+5=15$

Qus 89.

Let $X \subset \mathbb{R}^5$ be given the subspace topology. Which of the following statements are correct ?

(a) If X is finite, then every function $f : X \to \mathbb{R}$ is continuous.

(b) If every function $f : X \to \mathbb{R}$ is continuous, then X is finite

(c) If X is compact and infinite, then X is uncountable.

(d) If X is connected and has at least two elements, then X is uncountable.

Qus 90.

Let $\mathbb{R}$ denote the set of real numbers with euclidean topology. Let $\mathbb{R}_l$, denote the space of real numbers with lower limit topology. Recall that a basis of open sets for $\mathbb{R}_l$ is given by intervals of the form $[a, b)$ for all real numbers a, b.

Which of the following statements are correct ?

(a) If X is a nonempty connected subspace of $\mathbb{R}_l$, then X contains only one element.

(b) $\mathbb{R}_l$ is contains a countable dense subset

(c) Any open cover of $\mathbb{R}$ has a countable subcover.

(d) Any countable open cover of $\mathbb{R}$ has a finite subcover.

Qus 91.

Let $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ be two points on the $xy-$ plane with x_1 different from x_2 and $y_1 > y_2$.

Consider a curve

$C = \{z : z(x_1) = P_1, z(x_2) = P_2\}$. Suppose that a particle is sliding down along the curve C from the point P_1 to P_2 under the influence of gravity. Let T be the time taken to reach point P_2 and g denote the gravitational constant.

Which of the following statements are true ?

(a) $$T = \int_{x_1}^{x_2} \sqrt{\frac{1 + (z'(x))^2}{2 g z(x)}}\, dx$$

(b) $$T = \int_{x_1}^{x_2} \sqrt{\frac{1 + (z'(x))^2}{2 g z(x)}}\, dx$$

c) T is minimized when C is a straight line.

(d) The minimizer of T cannot be a straight line.

Qus. 92.

Let $X=\{u\in C^1[0,1]:u(0)=u(1)=0\}$. Let $I:X\to\mathbb{R}$ be defined as $I(u)=\int_0^1 e^{-u'(t)^2}\,dt$ for all $u\in X$.

Let $M=\sup_{f\in X} I[f]$ and $m=\inf_{f\in X} I[f]$.

Which of the following statements are true ?

(a) $M=1,\ m=0$

(b) $1=M>m>0$

(c) M is attained

(d) m is attained

Qus 93.

Let u be a solution of the following PDE

$$u_x+xu_y=0$$

$$u(x,0)=e^x$$

Which of the following statements are true ?

(a) $u(2,1)=e^2$

(b) $u(1,1/2)=1$

(c) $u(-2,1)=e^{-\sqrt{2}}$

(d) $u(-2,1)=e^{\sqrt{2}}$

Qus 94.

If $y(t)$ is a stationary function of

$J[y]=\int_{-1}^{1}(1-x^2)(y')^2\,dx$, $y(-1)=1$, $y(1)=1$

Subject to

$\int_{-1}^{1} y^2=1$

Which of the following statements are true ?

(a) y is unique

(b) y is always a polynomial of even order

(c) y is always a polynomial of odd order

(d) No such y exists

Qus 95.

Consider $\alpha=\dfrac{1+\sqrt{5}}{2}$ and $\beta=\dfrac{1-\sqrt{5}}{2}$. Define a sequence of numbers F_n as follows:

$F_n=\dfrac{\alpha^n-\beta^n}{\alpha-\beta}$ for $n=1,2,\ldots$

Let $p:\mathbb{R}\to\mathbb{R}$ be a polynomial of degree at most 2 such that $p(1)=F_1$, $p(3)=F_3$, $p(5)=F_5$

Which of the following statements are true ?

(a) $F_n=F_{n-1}+F_{n-2}$ for $n\geq 3$

(b) $p(7)=13$

(c) $F_n=F_{n-1}+2F_{n-2}$ for $n\geq 5$

(d) $p(7)=10$

Qus 96.

Let $u:\mathbb{R}^3\to\mathbb{R}$ satisfy $\Delta u=0$. Define $v(x)=u(Mx)$, where M is the 3×3 matrix. $M=\begin{pmatrix}0&-1&0\\1&0&0\\0&0&1\end{pmatrix}$

Which of the following statements are necessarily true ?

(a) $\Delta v=0$

(b) $\Delta v=v$

(c) $\dim(M^tM\,\nabla v)=0$

(d) $\dim(M^tM\,\nabla v)=v$

Qus 97.

Consider the ODE $\dot{x}=f(t,x)$ in $\mathbb{R}$, for a smooth function f.

Consider a general second order Runge-Kutta formula of the form

$x(t+h)=x(t)+w_1hf(t,x)+w_2hf(t+\alpha h,x+\beta hf)+O(h^3)$.

Which of the following choices of $(w_1, w_2, \alpha, \beta)$ are correct ?

(a) $\left(\frac{1}{2}, \frac{1}{2}, 1, 1\right)$ **(b)** $\left(\frac{1}{2}, 1, \frac{1}{2}, 1\right)$

(c) $\left(\frac{1}{4}, \frac{3}{4}, \frac{2}{3}, \frac{2}{3}\right)$ **(d)** $(0, 1, 1, 1)$

Qus 98.

Let g be the solution of the Volterra type integral equation $g(s) = 1 + \int_0^s (s-t) g(t)\,dt$; for all $s \geq 0$. What are the possible values of $g(1)$?

(a) $2e$ **(b)** $e - \frac{1}{e}$

(c) $e + \frac{1}{e}$ **(d)** $\frac{2}{e}$

Qus 99.

Consider the linear system $y' = Ay + h$ where $A = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}$ and $h = \begin{pmatrix} 3t+1 \\ 2t+5 \end{pmatrix}$ suppose $y(t)$ is a solution such that $\lim_{t\to\infty} \frac{y(t)}{t} = d \in \mathbb{R}^2$. What is the value of d ?

(a) $\begin{pmatrix} -\frac{4}{3} \\ -\frac{5}{3} \end{pmatrix}$ **(b)** $\begin{pmatrix} \frac{4}{3} \\ -\frac{5}{3} \end{pmatrix}$

(c) $\begin{pmatrix} -\frac{2}{3} \\ -\frac{5}{3} \end{pmatrix}$ **(d)** $\begin{pmatrix} \frac{2}{3} \\ -\frac{5}{3} \end{pmatrix}$

Qus 100.

Let $A \in M_3(\mathbb{R})$ be skew-symmetric and let $x : [0, \infty) \to \mathbb{R}^3$ be a solution of

$x'(t) = Ax(t)$, for all $t \in (0, \infty)$.

Which of the following statements are true ?

(a) $\|x(t)\| = \|x(0)\|$, for all $t \in (0, \infty)$

(b) For some $a \in \mathbb{R}^3 \setminus \{0\}$, $\|x(t) - a\| = \|x(0) - a\|$, for all $t \in (0, \infty)$

(c) $x(t) - x(0) \in \operatorname{im} A$, for all $t \in (0, \infty)$

(d) $\lim_{t\to\infty} x(t)$ exists

Qus 101.

Consider the two following initial value problems:-

(I) $y'(x) = y^{\frac{1}{3}}$, $y(0) = 0$

(II) $y'(x) = -y^{\frac{1}{3}}$, $y(0) = 0$

Which of the following statements are true ?

(a) I is uniquely solvable
(b) II is uniquely solvable
(c) I has multiple solutions
(d) II has multiple solutions

Qus 102.

Consider the following system of integral equations.

$$\varphi_1(x) = \sin x + \int_0^x \varphi_2(t)\,dt$$

$$\varphi_2(x) = 1 - \cos x - \int_0^x \varphi_1(t)\,dt$$

Which of the following statements are true ?

(a) φ_1 vanishes at atmost countably many points

(b) φ_1 vanishes at uncountably many points

(c) φ_2 vanishes at atmost countably many points

(d) φ_2 vanishes at uncountably many points

Qus 103.

The observation X has normal distribution with unknown mean $-\infty < \mu < \infty$ and variance 1. Consider the problem of estimation of μ based on X under the squared error loss. Which of the following statements are correct ?

(a) The Bayes estimator of μ under a proper prior is always biased.

(b) There is a proper prior for which the Bayes estimator of μ is unbiased.

(c) The Bayes estimator of μ under an improper prior is always biased.

(d) There is an improper prior for which the Bayes estimator of μ is unbiased.

Qus 104.

Let X be a random variable whose probability mass functions under H_0 and H_1 are given by the following.

x	1	2	3	4	5	6	7
$f_{H_0}(x)$	0.01	0.01	0.01	0.01	0.01	0.01	0.94
$f_{H_1}(x)$	0.06	0.05	0.04	0.03	0.02	0.01	0.79

where $f_{H_0}(x) = P_{H_0}(X = x)$ and $f_{H_1}(x) = P_{H_1}(X = x)$. Which of the following statements are correct ?

(a) The critical region of most powerful test of size $\alpha = 0.04$, for testing H_0 against H_1, is given by $\{x : x \le 4\}$

(b) The critical region of most powerful test of size $\alpha = 0.04$, for testing H_0 against H_1, is given by $\{x : 3 \le x \le 6\}$

(c) The power of the most powerful test of size $\alpha = 0.04$, for testing H_0 against H_1 is 0.18

(d) Most powerful test of size $\alpha = 0.04$, for testing H_0 against H_1 does not exist.

Qus 105.

Suppose $X_1, ..., X_n$ are independent and identically distributed random variables from the normal distribution with mean θ and known variance σ^2. If the prior distribution of θ is normal with mean μ and variance τ^2, then which of the following statements are correct ?

(a) With respect to the squared error loss function., Bayes estimator is the mean of the posterior distribution.

(b) With respect to the squared error loss function, Bayes estimator is the median of the posterior distribution.

(c) With respect to the absolute error loss function, Bayes estimator is the median of the posterior distribution.

(d) With respect to the absolute error loss function, Bayes estimator is the mean of the posterior distribution.

Qus 106.

Let $\{X_i ; i \ge 1\}$ be a sequence of independent and identically distributed Bernoulli random variables with $\mathbf{P}(X_1 = 1) = p \in (0,1)$. Which of the following sequences of estimators are consistent for p as $n \to \infty$?

(a) $\left\{\frac{1}{n} \sum_{i=1}^{n} X_i : n \ge 1\right\}$

(b) $\{0.5(X_n + X_{n+1}) : n \ge 1\}$

(c) $\left\{\frac{1}{n} \sum_{i=1}^{n} X_i^i : n \ge 1\right\}$

(d) $\left\{\frac{1}{n} \sum_{i=1}^{n} X_{2i-1} X_{2i} : n \ge 1\right\}$

Qus 107.

Let $\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \sim N_3\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho & 0 \\ \rho & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right)$ where $|\rho| < 1$. Which of the following are true ?

(a) $Cor(X^2, Y^2) = \rho^2$

(b) $Cor(X^2, Y) = 0$

(c) $Cor(X^2, Z^2) = 0$

(d) $Cor(X^2, Y^2 + Z^2) = \rho^2$

Qus 108.

Consider the hierarchical single linkage agglomerative clustering algorithm for three points $X = (1,0), Y = (0,2), Z = (3,3)$, and the squared Euclidean distance matrix. The clustering algorithm starts with three clusters. Then which of the following statements are correct.

(a) At the first step, after combining two nearest clusters, the single linkage distance between the two newly formed clusters is 10.

(b) At the first step, X and Y are merged to form a new cluster.

(c) At the first step, after combining two nearest clusters, the single linkage distance between the two newly formed clusters is 13.

(d) At the first step, X and Z are merged to form a new cluster.

Qus 109.

Let $\{X_n : n \geq 1\}$ be a sequence of independent and identically distributed random variables and the probability mass function of X_1 is the following $P(X_1 = 1) = P(X_1 = 3) = \frac{1}{2}$.

If $Y_n = X_1 + ... + X_n$, then which of the following statements are correct ?

(a) $\frac{Y_n}{n}$ converges to 2 in probability.

(b) Variance $\left(\frac{Y_n}{n^{2/2}}\right)$ converges to 0, as $n \to \infty$

(c) $\frac{Y_n}{n^{2/2}}$ converges to c in probability, where $0 < c < \infty$

(d) $\frac{Y_n}{n^2}$ converges to 0 in probability

Qus 110.

Let X be a random variable whose distribution is symmetric about -2. Which of the following are true ?

(a) If X discrete and $P(X = -2) = \frac{2}{3}$, then $P(X > -2) = 1/6$

(b) If X is discrete and $P(X = -2) = \frac{1}{2}$, then $P(X > -2) = 1/2$

(c) If X is absolutely continuous, then $P(X > -2) = 0$

(d) If X is absolutely continuous, then $P(X > -2) = 1/2$

Qus 111.

Let X_1 and X_2 be independent and identically distributed standard normal variables. Then which of the following statements are correct ?

(a) Expected value of $\max(X_1, X_2)$ is $\frac{1}{\sqrt{\pi}}$

(b) Conditional expectation of X_1 given $X_1 + X_2$ is $0.5(X_1 + X_2)$

(c) $X_1 - X_2$ and $X_1 + X_2$ are independent

(d) $X_1^2 + X_2^2$ and $\frac{X_1}{X_2}$ are independent

Qus 112.

Let $X_1, X_2, ..., X_n$ be independent and identically distributed normal random variables with mean μ and variance σ^2. Suppose $\left(\hat{\mu}, \hat{\sigma}^2\right)$ is the maximum likelihood estimator of (μ, σ^2). Which of the following statements are correct ?

(a) $\frac{(n-1)\hat{\sigma}^2}{\sigma^2}$ is a chi-square random variable with $(n-1)$ degrees of freedom.

(b) $\hat{\sigma}^2$ is an unbiased estimator of σ^2

(c) Variance of $\hat{\sigma}^2$ tends to 0 as $n \to \infty$

(d) $\frac{n^2 \hat{\sigma}^4}{(n-1)(n+1)}$ is an unbiased estimator of σ^4

Qus 113.

Consider the multiple linear regression model $Y_1 = \beta_1 x_{i1} + ... + \beta_p x_{ip} + \in_i$, with $E(\in_i) = 0$, $Cov(\in_i, \in_k) = 0$, If $i \neq k$ and $Var(\in_i) = \sigma^2$, for $i, k = 1, ..., n$.

If $\widehat{y_i}$ is the least squares fit of y_i and $\widehat{\in_i}$ is the corresponding estimated residual for $i = 1,...,n$ then which of the following statements are always correct ?

(a) $\sum_i \widehat{\in_i} = 0$ and $\sum_i \widehat{y_i}\widehat{\in_i} = 0$

(b) $\sum_i x_{ij}\widehat{\in_i} = 0$; for all $j = 1,..., p$

(c) $\sum_i \widehat{\in_i} = 0$ and $\sum_i x_{ij}\widehat{\in_i} = 0$; for all $j = 1,..., p$

(d) $\sum_i \widehat{y_i}\widehat{\in_i} = 0$

Qus 114.
If the hazard function of a lifetime random variable X is given by

$r(x) = \dfrac{2x}{1+x^2}, x \in (0,\infty)$, indicate the correct options.

(a) Survival function of the random variable X is $S(x) = \dfrac{1}{1+x^2}, x \in (0,\infty)$

(b) Survival function of the random variable X is $S(x) = \dfrac{x^2}{1+x^2}, x \in (0,\infty)$

(c) Survival function of the random variable X cannot be determined from the given information

(d) The survival function, density function and distribution function can be determined from the given information

Qus 115.
The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Let $Y = [X]$, where $[X]$ denotes the largest integer not exceeding X. Which of the following statements are correct ?

(a) $P(Y = 2) = 0$

(b) $P(Y < 1.2) = 1 - e^{-2}$

(c) $E(Y^2) = \dfrac{e+1}{(e-1)^2}$

(d) $E(Y) = \dfrac{1}{(e-1)}$

Qus 116.
Let

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

be the one step transition probability matrix of a stationary Markov Chain. Which of the following statements are true ?

(a) All the states have same periods
(b) All the states are transient
(c) Some states are transient
(d) All the states are recurrent

Qus 117.

Let $X_1, X_2, ..., X_{10}$ be a random sample from *Uniform* $(0,1)$ and $X_{(1)}, X_{(2)}, ..., X_{(10)}$ denote the corresponding order statistics. Which of the following statements are true ?

(a) $X_{(2)} \sim Beta(2,9)$

(b) $X_{(10)} - X_{(1)} \sim Beta\,(11,2)$

(c) $E\left[X_{(2)}\right] = \dfrac{2}{11}$

(d) $Var\left[X_{(2)}\right] = \dfrac{3}{242}$

Qus 118.

Let $X_1, ..., X_{10}$ be a random sample of size 10 from a continuous distribution with probability density function

$$f_\theta(x)=\begin{cases} e^{(\theta-x)}, & x\ge\theta \\ 0 & \text{otherwise} \end{cases} \quad \theta\in R \text{ is unknown.}$$

Consider the problem of testing the null hypothesis $H_0:\theta\le 3$ against the alternate hypothesis $H_1:\theta>3$, based on $X_1,\ldots,X_{10}$. Let $L(\theta)$ denote the likelihood function and $x_{(1)}=\min(x_1,\ldots,x_{10})$

Which of the following statements are correct ?

(a) $\sup\limits_{\theta\le 3} L(\theta)=e^{10x_{(1)}}\,e^{-\sum_{i=1}^{10}x_i}$, If $x_{(1)}\le 3$

(b) $\sup\limits_{\theta\le 3} L(\theta)=e^{10x_{(1)}}\,e^{-\sum_{i=1}^{10}x_i}$, If $x_{(1)}>3$

(c) The critical region of the likelihood ratio test of size $\alpha\,(0<\alpha<1)$, for testing H_0 against H_1, is given by

$$\left\{(x_1,\ldots,x_{10})\in R^{10}: x_{(1)}\ge\frac{1}{3}-10\log_e(\alpha)\right\}$$

(d) The critical region of the likelihood ratio test of size $\alpha\,(0<\alpha<1)$, for testing H_0 against H_1, is given by

$$\left\{(x_1,\ldots,x_{10})\in R^{10}: x_{(1)}\ge 3-\frac{1}{10}\log_e(\alpha)\right\}$$

Qus 119.

Let

$$P=\begin{bmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$$

be the one step transition probability matrix of a homogeneous Markov Chain. Which of the following statements are true ?

(a) It is an irreducible markov chain

(b) All the states are recurrent..

(c) $\lim\limits_{n\to\infty}P^n$ exists

(d) All the states have same period

Qus 120.

Consider the simple linear model $Y_i=\beta X_i+\in_i$, for $i=1,\ldots,n$ where $\in_i$'s are i.i.d $N(0,\sigma^2)$ random variables and X_i's are nonrandom, postive and distinct. Consider two estimates of β given below

$$\widehat{\beta_1}=\frac{\sum_{i=1}^{n}(Y_i-\overline{Y})(X_i-\overline{X})}{\sum_{i=1}^{n}(X_i-\overline{X})^2}$$

$$\widehat{\beta_2}=\frac{\sum_{i=1}^{n}Y_iX_i}{\sum_{i=1}^{n}X_i^2}$$

Which of the following are correct ?

(a) Both $\widehat{\beta_1}$ and $\widehat{\beta_2}$ are unbiased estimates for β

(b) $\widehat{\beta_1}$ has larger variance then $\widehat{\beta_2}$

(c) $\widehat{\beta_2}$ has larger variance then $\widehat{\beta_1}$

(d) $\widehat{\beta_1}$ has the same variance as that of $\widehat{\beta_2}$

CSIR-NET EXAM ANSWER KEY

16- SEPTEMER -2022

Q. 1 (D) Q. 2 (D) Q. 3 (A) Q.4 (C) Q. 5 (C) Q. 6 (C) Q. 7 (B)

Q. 8 (A) Q. 9 (B) Q. 10 (D) Q. 11 (A) Q. 12 (B) Q. 13 (B) Q. 14 (A)

Q.15 (D) Q. 16 (A) Q. 17 (B) Q. 18 (C) Q. 19 (B) Q. 20 (D) Q. 21 (B)

Q.22 (C) Q. 23 (D) Q. 24 (B) Q. 25 (C) Q. 26 (C) Q. 27 (B) Q. 28 (B)

Q.29 (D) Q. 30 (A) Q. 31 (D) Q. 32 (C) Q. 33 (B) Q. 34 (B) Q. 35 (D)

Q.36 (D) Q. 37 (C) Q. 38 (C) Q. 39 (D) Q. 40 (D) Q. 41 (A) Q. 42 (C)

Q.43 (A) Q. 44 (A) Q. 45 (A) Q. 46 (C) Q. 47 (C) Q. 48 (C) Q. 49 (D)

Q.50 (B) Q. 51 (C) Q. 52 (D) Q. 53 (C) Q. 54 (D) Q. 55 (A) Q. 56 (C)

Q.57 (C) Q. 58 (B) Q. 59 (A) Q.60 (D) Q. 61 (A,B,D) Q. 62 (A,C)

Q. 63 (D) Q. 64 (A,D) Q. 65 (A,D) Q. 66 (D) Q. 67 (A,C,D) Q. 68 (C)

Q. 69 (C) Q. 70 (D) Q. 71 (A,C) Q. 72 (B,C) Q. 73 (C) Q. 74 (A,B) Q. 75 (A,D)

Q. 76 (A,C,D) Q. 77 (B,D) Q. 78 (A,C,D) Q. 79 (A,D) Q. 80 (A,C) Q. 81 (A,B,C)

Q. 82 (A,B,C) Q. 83 (A,B,D) Q. 84 (A,B) Q. 85 (A,B,C,D) Q. 86 (A,C) Q. 87 (B,C)

Q. 88 (A,C) Q. 89 (A,D) Q. 90 (A,B,C) Q. 91 (A,D) Q. 92 (A,C) Q. 93 (B,C)

Q. 94 (D) Q. 95 (A,D) Q. 96 (A,C) Q. 97 (A,C) Q. 98 (A) Q. 99 (A) Q. 100 (A,B,C)

Q. 101 (B,C) Q. 102 (A,D) Q. 103 (A,D) Q. 104 (A,C) Q. 105 (A,B,C,D) Q. 106 (A,C)

Q. 107 (A,B,C) Q. 108 (A,B) Q. 109 (A,B,D) Q. 110 (A,D) Q. 111 (A,B,CD)

Q. 112 (C,D) Q. 113 (B,D) Q. 114 (A,D) Q. 115 (B,C,D) Q. 116 (A,C) Q.117 (A,C,D)

Q. 118 (A,D) Q. 119 (B,D) Q. 120 (A,B)

CSIR NET SOLUTION
16-Sept-2022

PART "A"

Qus. 1
Sol:- **(d)** From the given bar chart it is clear that A has highest market share per employee (around 0.9) and B has highest sale for it's market share (again around 0.9)

Qus. 2
Sol:- **(d)** Let E,H, & S be the number of elephants, horses and soldiers then

$E+H+S=3500$ (i)

$H=2E$ (ii)

& $4E+4H+\left(\frac{3}{4}S\right)\times 2 = 7500$ (iii)

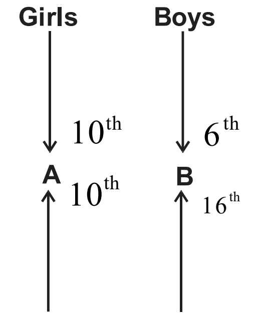
Now $(i)\times 4-(iii)$ $\Rightarrow$

$\frac{5}{2}S=6500 \Rightarrow S=2600$

$\Rightarrow$ $E+H=3500-2600 \Rightarrow \frac{1}{2}H+H=900$

$\Rightarrow$ $H=600$ Ans.

Qus. 3
Sol:- **(b)**

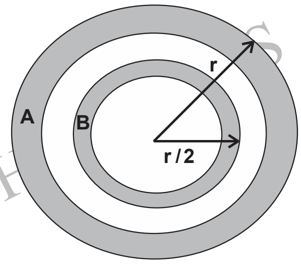


$\underline{9G+5B}$ AB $\underline{9G+15B}$

From the above picture rank of A from above is $9+5+1=15^{th}$

and from bottom it is $9+15+1+1=26^{th}$ i.e. 15^{th} from top & 26^{th} from bottom.

Qus. 4
Sol:- **(c)**



Ratio of area of track A to that of area of track B is

$$\frac{\pi\left(30^2-(28)^2\right)}{\pi\left(15^2-13^2\right)}=\frac{30+28}{15+13}=\frac{58}{28}$$

$$=\frac{29}{14}$$

Qus. 5
Sol:- **(c)** Based on the above experiment we can conclude that there is a decrease in pH and an increase in both T and PCO_2 over 20 days

Qus. 6
Sol:- **(c)** Out of $15\times 6=90$ seats $15\times 2=30$ seats are on either sides so probability of getting seat on either side is

$$P(E)=\frac{30}{90}=\frac{1}{3}$$

Qus. 7
Sol:- **(b)** If x more incorrect answers were given by A then

$$(x)\times\left(-\frac{1}{4}\right)+(5-x)\times(1)=0$$

$\Rightarrow \quad 5=\frac{5x}{4} \Rightarrow x=4$

Qus. 8

Sol:- **(a)** If graph of logarithm is converted into linear it will look like that in picture A.

Qus. 9

Sol:- **(b)** In 5 kg. Watermelon solid stuff is 1% = 0.05kg. and if 98% is water then weight of 2% watermelon is 0.05kg. $\Rightarrow$ weights of watermelon $=\frac{0.05}{2}\times 100=2.5$ kg.

Qus. 10

Sol:- **(d)** Of the given statements , statement 4 is true and rest are false

Qus. 11

Sol:- **(a)** $\because 27=(3)^3$ so minimum number of weighings required to indentity dummy coin is $3=\log_3 27$

Qus. 12

Sol:- **(b)** Cost of 120 units of items =

$$C_0\times\frac{90}{100}\times\frac{90}{100}\times 120 = 97.2\ C_0 = 97C_0$$

Qus. 13

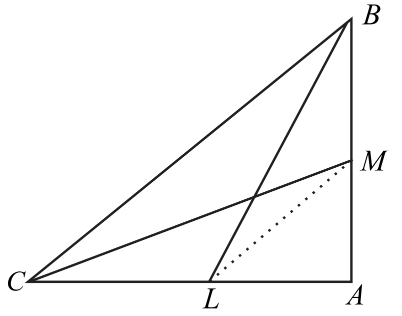
Sol:- **(b)** Percentage of alcohol after adding 3600 ml of water =

$$100\times\frac{360\times\frac{40}{100}}{360+3600}=\frac{14400}{3960}=\frac{1800}{495}=\frac{200}{55}$$

$=\frac{40}{11}=3.6$ % "approximately"

Qus. 14

Sol:- **(a)**



$$\frac{BC}{ML}=\frac{2}{1}=2$$

because length of line segment joining mid-point of 2 sides of a triangle is parallel to the third side and it is of half length than it.

Qus. 15

Sol:- **(d)** As arithmetic mean of 5 numbers is zero so sum of those 5 numbers is also zero hence their cannot be exactly one non-zero number in it else their sum will be that non-zero number.

Qus. 16

Sol:- **(a)** As magazine M is read by the highest number of readers so M is the most popular english magazine published in the country

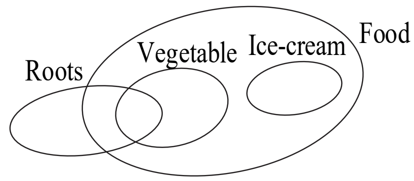
Qus. 17

Sol:- **(None are correct)**

According to the given information second digital clock is either not moving or it has moved 24 hours in 30 seconds so their is no concrete information about it

Qus. 18

Sol:- **(c)**



Venn Diagram

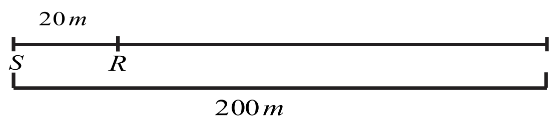
Qus. 19

Sol:- **(a)** If third digit is K ; $K\in\{0,1,2,\ldots 9\}$ then we have $K+1$ options so number of such three digit PINS are equal to

$$\sum_{K=0}^{9}(K+1)=1+2+\ldots+10=\frac{10\times 11}{2}=55$$

Qus. 20

Sol:- **(d)**



R run 180 m in $\frac{200}{10}=20$ second

So, R will run $200-180=20$ meter in

$\frac{20}{180}\times 20$ second $=\frac{20}{9}$ second =
2.2 seconds (approximately)

PART "B"

Qus. 21.

Sol:- **(b)** If Quadratic form $Q(x_1,x_2,x_3)$ is positive definite then all square terms will have positive coefficient, so if $Q(x_1,x_2,x_3)=X^t AX$ then $a_{ii}>0$; $\forall i=1,2,3$

Qus. 22.
Sol:- **(c)**

$\because \limsup\left((-1)^n b_n\right)\le |\limsup b_n|+|\liminf b_n|$

$\therefore \limsup\left(a_n+(-1)^n b_n\right)\le$

$\limsup a_n+|\limsup b_n|+|\liminf b_n|$

Qus. 23.

Sol:- **(d)** $a_n=n+\frac{1}{n} \Rightarrow \sum(-1)^{n+1}\frac{a_{n+1}}{n!}=$

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\left[(n+1)+\frac{1}{(n+1)}\right]}{n!}=$$

$$\sum_{n=1}^{\infty}(-1)^{n+1}\left[\frac{1}{(n-1)!}+\frac{1}{n!}+\frac{1}{(n+1)!}\right]$$

$$=\left[1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\ldots\right]$$

$$+\left[\frac{1}{1!}-\frac{1}{2!}+\frac{1}{3!}-\frac{1}{4!}+\ldots\right]$$

$$+\left[\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\ldots\right]$$

$$=\left(e^{-1}\right)+\left(1-e^{-1}\right)+e^{-1}$$

$$=e^{-1}+1$$

Qus. 24

Sol:- **(b)** In Fibonacci like sequence $a_1,a_2\in R$ but $\forall\ n\ge 3$, $a_n=a_{n-1}+a_{n-2}$

So, there are two arbitrary constants. Hence dimension of vector space is 2

Qus. 25

Sol:- **(c)** $\langle ax+b,\ x+c\rangle=\int_0^1(ax+b)(x+c)\,dx$

$$=\frac{a}{3}+\frac{ac+b}{2}+bc$$

so, $\langle ax+b,x+c\rangle=0 \Rightarrow$

$$\left(\frac{a}{2}+b\right)c=-\frac{a}{3}-\frac{b}{2} \Rightarrow$$

$\forall\ c\in R^+$ there will be an equation in 2 variables a & b whose number of solutions is infinite.

Qus. 26

Sol:- **(c)** For series $\sum_{n=3}^{\infty}\frac{a^n}{n^b(\log_e n)^c}$

If $a=1$; $0\le b\le 1$ and $c<1$ then series is $\sum_{n=3}^{\infty}\frac{1}{n^b(\log_e n)^c}$ diverges to ∞, so it is not convergent.

Qus. 27

Sol:- **(d)** $X=\left\{(x_n)_{n\ge 1}\ ;\ \limsup_{n\to\infty}=1\ ;\ x_n\in\{0,1\}\right\}$ is uncountable set as after any finite therms & upto infinite terms x_n can take both values 0 & 1
Also

$$Y=\left\{(x_n)_{n\ge 1}\ ;\ \lim_{n\to\infty}x_n \text{ does not exist }; x_n\in\{0,1\}\right\}$$

is uncountable set as both 0 & 1 are possible value of x_n upto infinite terms.

Qus. 28

Sol:- **(b)** $f_n(t)=(n+2)(n+1)t^n(1-t)$

$\Rightarrow \quad f(t)=\lim_{n\to\infty}f_n(t)=0\ ;\ \forall t\in[0,1]$

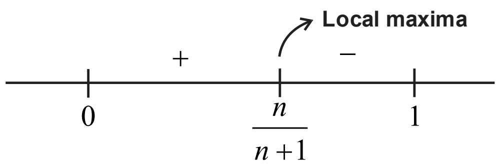
$$\left|f_n(t)-f(t)\right| = (n+2)(n+1)t^n(1-t)$$

$= g(t)$ (let)

$$g'(t) = (n+2)(n+1)\left[-t^n + n\,t^{n-1}(1-t)\right]$$

$$= (n+2)(n+1)t^{n-1}\left[n-(n+1)t\right]$$

$$= -(n+2)(n+1)^2 t^{n-1}\left[t-\frac{n}{n+1}\right]$$



Sign scheme of g'(t)

So, $\sup\limits_{t\in[0,1]}\left|f_n(t)-f(t)\right| = \sup\limits_{t\in[0,1]} g(t) = g\left(\frac{n}{n+1}\right) =$

$$(n+2)(n+1)\frac{n^n}{(n+1)^n}\times\frac{1}{(n+1)}$$

$\Rightarrow$ $M_n = (n+2),\dfrac{1}{\left(1+\frac{1}{n}\right)^n}$ & $\lim\limits_{n\to\infty} M_n = \infty$, so convergence is not uniform, it is only pointwise.

Qus. 29

Sol:- (d) If A is similar to B then $A^T \sim A$ & $B^T \sim B$

so $A^T \sim B^T$ (1)

Minimal polynomial of $A =$
Minimal polynomial of B (2)
Trace (A) = Trace (B)
but Range(A) need not be equal to Range(B)

$\because$ $\text{Range}\left(\begin{bmatrix}1 & 4\\ 0 & 2\end{bmatrix}\right) \neq \text{Range}\left(\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}\right)$

Qus. 30

Sol:- (a) $C(A) = C(A^{-1}) \Rightarrow$ If λ is an eigen value of A then $\dfrac{1}{\lambda}$ is also an eigenvalue of A.

So, 1 eigenvalue of A will be self inverse i.e. 1 or -1

So, $\det(A) = 1$ *or* -1

& $\det(A)^2 = 1$

Qus. 31

Sol:- (d) $V = \{A\in M_n(R) \mid AX = 0\}$

$\Rightarrow$ $\dim(V) = n(n-\text{Rank}(A))$

$= n(n-r)$

$= n^2 - nr$

Qus. 32

Sol:- (c) $B_n(p) = \left\{x\in D\ ;\ d(x,p) < \frac{1}{n}\right\}$

$$W_p = \bigcap_n \overline{f(B_n(p))}$$

$$= \lim_{n\to\infty}\overline{f(B_n(p))} = \{p\}$$

$\Rightarrow$ W_p is singleton for every p.

Qus. 33

Sol:- (b) $\left|e^{e^z}\right| = 1 \Rightarrow \left|e^{e^{x+iy}}\right| = 1$

$\Rightarrow$ $\left|e^{e^x(\cos y + i\sin y)}\right| = 1$

$\Rightarrow$ $\left|e^{e^x\cos y}.\, e^{ie^x\sin y}\right| = 1$

$\Rightarrow$ $e^{e^x\cos y} = 1$

$\Rightarrow$ $e^x\cos y = 0 \quad \Rightarrow \cos y = 0$

$\Rightarrow$ $y = (2n+1)\dfrac{\pi}{2}\ ;\ n\in I$

Qus. 34

Sol:- (b)

$R \to CRU$

$S \cap I = \phi$

Let $R = \mathbb{Z}$

$I = \langle 2\rangle = \{2n : n\in\mathbb{Z}\}$

$S = \{3^n : n\in\mathbb{Z}\}$

$I = \langle 1\rangle = \mathbb{Z}$ but $I = 2\mathbb{Z}$

$\therefore$ (c) discard

$S \cap I = \phi$

I is maximal ideal w.r.t

$S \cap I = \phi$ i.e. $I \subseteq I_1$ then

$S \cap I_1 \neq \phi$

$R = \mathbb{Z},\ I = \langle 2 \rangle, S = 3^n$

$I \subseteq I_1$

$\exists\ a \in \mathbb{Z}$ such that

a is odd $a \in I_1$

$\therefore\ a+1$ even

$a+1 \in I$ i.e $a+1 \in I_1$

$\therefore\ a+1-a = 1 \in I_1$

$\therefore\ I_1 = R$

$I = \langle 0 \rangle = \langle 0 \rangle$ but $I = \langle 2 \rangle$

$\therefore$ (d) discard

Now w.r.t in CRU every maximal ideal is prime ideal

$\therefore$ (a) discard

Qus. 35

Sol:- (d)

$N = \{x \in R \mid x^n = 0 \text{ for some } n \in \mathbb{N}\}$

Clearly $\phi \neq N \subseteq R\ (0 \in N)$

Ideal if

i) $x, y \in N$ s.t. $x - y \in N$

ii) $x \in N\ a \in R$ s.t. $a\,x \in N$

Let $x^n = 0$ and $y^m = 0$. for $m, n \in \mathbb{N}$

If R is commutative then

$(x-y)^{n+m} = {}^{n+m}C_0 x^{n+m} y^0 - {}^{n+m}C_1 x^{n+m-1} y^1$

$+ \ldots .. + {}^{n+m}C_m x^n y^m + {}^{n+m}C_{m+1} x^{n-1} y^{m+1}$

$+ \ldots . + y^{n+m}$

$x^n = 0$ then $x^t = 0\ \ \forall\ t \geq n$

$y^m = 0$ then $y^t = 0\ \ \forall\ t \geq m$

$\therefore\ (x-y)^{n+m} = 0$

$\therefore\ x - y \in N$

ii) $x \in N$ and $a \in N$

$x^n = 0$ for some $n \in N$

If R is commutative then

$(ax)^n - a^n x^n = 0$

$\Rightarrow\ (x\,a)^n = x^n\,a^n = 0$

$\therefore$ (d)

Qus. 36

Sol:- (d) $f(z) = (1-z)e^{\left(z+\frac{z^2}{2}\right)}$

$= 1 + \sum_{n=1}^{\infty} a_n z^n$

$\Rightarrow\ f'(z) = \left[(1-z)(1+z) - 1\right] e^{\left(z+\frac{z^2}{2}\right)}$

$= -z^2\, e^{\left(z+\frac{z^2}{2}\right)}$

Now, $f(z) = (1-z)e^z \left(e^{\frac{z^2}{2}}\right)$

$= (1-z)\left[1 + \frac{z}{1!} + \frac{z^2}{2!} + \ldots\right]\left[1 + \frac{z^2}{2} + \frac{z^4}{2!(2)^2} + \ldots\right]$

Hence $a_1 = -1 + \frac{1}{1!} = 0$

$a_2 = \frac{1}{2} + \frac{1}{2} - 1 = 0$

$\Rightarrow\ a_1 = a_2$

Also, $\forall\ n \in N\ ;\ a_n \in (-\infty, 0]$

$\because$ Coefficient of $\mathbb{Z}^n$ has positive term $\leq$ negative term

"so, 4th is false"

Qus. 37

Sol:- (c) If X is connected metric space with at least two points then X must be uncountable set.

Qus. 38

Sol:- (c) $168 = 7 \times 24$

$1 + 7.1 = 8$ divides 168

So, there are 8 subgroups of order 7.

Hence number of elements of order 7 =

$(7-1) \times 8 = 48$

Qus. 39

Sol:- (d) Method 1:-

$$f^{(2)}+f=0 \quad \Rightarrow \ f^{(2)}=-f$$

$$\Rightarrow \quad f^{(4)}=-f^{(2)} \Rightarrow f^{(6)}=-f^{(4)}....$$

$$\Rightarrow \quad f=f^{(4)}=f^{(8)}=f^{(12)}=....$$

$$=-f^{(2)}=-f^{(6)}=-f^{(10)}...$$

$$\& \ f'=f^{(5)}=f^{(9)}=.... \quad =$$

$$-f^{(3)}=-f^{(7)}=...$$

Hence $\left|f^{(n)}(0)\right|=\left|f(0)\right|$ if n is even

$=\left|f'(0)\right|$ if n is odd

So, $\left|f^{(n)}(0)\right|$ has convergent subsequences

Method 2:

$$f^{(2)}+f=0 \quad \Rightarrow \left(D^2+1\right)f=0 \Rightarrow$$

$$f(z)=c_1\cos z+c_2\sin z$$

$$\Rightarrow \quad f^n(0)=c_1\cos\left(\frac{n\pi}{2}\right)+c_2\sin\left(\frac{n\pi}{2}\right)$$

$$\Rightarrow \quad \left|f^n(0)\right|=\left|c_2\right|; \ \textit{if } n \textit{ is odd}$$

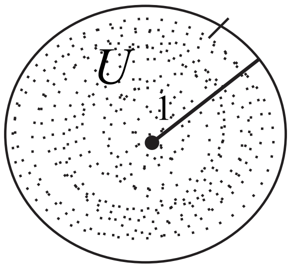
$$=\left|c_1\right|; \ \textit{if } n \textit{ is even}$$

So, it has $\{S_{odd}\}$ & $\{S_{even}\}$ as convergent sub-sequences.

Qus. 40

Sol:- (d) $f(z)$ is non constant entire function such that $\left|f(z)\right|=1$ for $|z|=1$

$$U=\left\{z\in C \mid |z|<1\right\}$$



According to the condition given

$f(z)=z^n \ ; \forall \ z\in c$

So, (i) $f(c)=c$

(ii) $f(z)=0 \Rightarrow z=0$, so $f(z)$ has at least one zero in U

(iii) f has at most finitely many distinct zeroes in C

but $f(z)$ cannot have a zero outside C

Qus. 41

Sol:- (a)

Qus. 42

Sol:- (c)

$a u_{xx}+b u_{xy}+a u_{yy}=0$ in $\mathbb{R}^2$

$S=b \quad r=a \quad t=a$

$b^2-4a^2<0 \quad \Rightarrow$ PDE is elliptic

$b^2<4a^2$

$|b|<2|a| \Rightarrow$ PDE is elliptic

Qus. 43

Sol:- (a)

For any two continuous functions $f,g:R\to R$ define

$$f*g(t)=\int_0^t f(s)g|(t-s)ds$$

$f*g(t)$ when $f(t)=e^{-t}$, $g(t)=\sin t$

$$\Rightarrow \quad f*g(t)=\int_0^t e^{-s}\sin(t-s)ds$$

$$=f*g(t)=\int_0^t e^{-s}\left(\sin t\cos s-\cos t\sin s\right)ds$$

$$=f*g(t)=\sin t\int_0^t e^{-s}\cos s\,ds-\cos t\int_0^t e^{-s}\sin s\,ds$$

$$\Rightarrow \quad \int_0^x e^{-x}\cos x=\frac{e^{-x}}{2}\left[-\cos x+\sin x\right]$$

$$\int_0^x e^{-x}\sin x=-\frac{e^{-x}}{2}\left[\sin x+\cos x\right]$$

$$\Rightarrow \quad \sin t\left\{\frac{e^{-s}}{2}\left[\sin s-\cos s\right]\right\}_0^t-$$

$$\cos t\left\{-\frac{e^{-s}}{2}\left[\sin s+\cos s\right]\right\}_0^t$$

$$= \sin t\left[\frac{e^{-t}}{2}\left[\sin t - \cos t\right] + \frac{1}{2}\right] -$$

$$\cos t\left[-\frac{e^{-s}}{2}\left(\sin t + \cos t\right) - \frac{1}{2}\right]$$

$$\Rightarrow \quad \frac{e^{-t}}{2}\left[\sin^2 t - \sin t\cos t + \frac{1}{2}\sin t + \sin t\cos t + \right.$$

$$\left.\cos^2 t - \frac{1}{2}\cos t\right]$$

$$\Rightarrow \quad \frac{e^{-t}}{2}\left[1 + \frac{1}{2}\sin t - \frac{1}{2}\cos t\right]$$

$$\frac{1}{2}\left[e^{-t} + \sin t - \cos t\right]$$

Qus. 44
Sol:-(a)

Given hyperbola $xy = b$ (1)

$$\Rightarrow \quad y = \frac{b}{x}$$

Lagranagian (L) given by

$L = T - V$ $\quad T \to KE$

$V \to PE$

$T = \frac{1}{2}MV^2 = \frac{1}{2}Mx^{\bullet 2}$ (one-dimensional)

Differentiable (1) w.r.t time t

$$\therefore \quad \frac{d}{dt}(xy) = \frac{d}{dt}(b)$$

$$\Rightarrow \quad x^{\bullet}y + y^{\bullet}x = 0$$

$$y^{\bullet} = \frac{-x^{\bullet}y}{x}$$

$$\therefore \quad T = \frac{1}{2}M\left(x^{\bullet 2} + \frac{x^{\bullet 2}y^2}{x^2}\right)$$

$$T = \frac{1}{2}M\left(x^{\bullet 2} + \frac{x^{\bullet 2}}{x^2}\left(\frac{b}{x}\right)^2\right)$$

$$T = \frac{1}{2}M\left(x^{\bullet 2} + \frac{x^{\bullet 2}b^2}{x^4}\right)$$

$$T = \frac{1}{2}M\,x^{\bullet 2}\left(1 + \frac{b^2}{x^4}\right)$$

$$V = m\,g\,y$$

$$V = \frac{m\,g\,b}{x}$$

$$L = \frac{1}{2}Mx^{\bullet 2}\left(1 + \frac{b^2}{x^4}\right) - \frac{-m\,g\,b}{x}$$

Equation of motion is given by (of a particle)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial x^{\bullet}}\right) - \frac{\partial L}{\partial x} = 0 \qquad (2)$$

$$\Rightarrow \quad \frac{d}{dt}\left[\frac{\partial}{\partial x^{\bullet}}\left(\frac{1}{2}M\,x^{\bullet 2}\left(1 + \frac{b^2}{x^4}\right) - \frac{mgb}{x}\right]\right.$$

$$\left. - \frac{\partial}{\partial x}\left[\frac{1}{2}Mx^{\bullet 2}\left(1 + \frac{b^2}{x^4}\right) - \frac{mgb}{x}\right] = 0\right.$$

$$\Rightarrow \quad \frac{d}{dt}\left(mx^{\bullet}\left(1 + \frac{b^2}{x^4}\right)\right) - \left[\left(\frac{1}{2}mx^{\bullet 2}\left(\frac{-4}{x^5}b^2\right)\right.\right.$$

$$\left. + \frac{mgb}{x^2}\right] = 0$$

$$\Rightarrow \quad m\,x^{\bullet\bullet}\left(1 + \frac{b^2}{x^4}\right) + \frac{2\,mx^{\bullet 2}b^2}{x^5} - \frac{mgb}{x^2} = 0$$

$\therefore$ (a)

Qus. 45
Sol:- (a)

Qus. 46
Sol:- (c)

Result

(i) If U(x,t) is solution of wave equation then U(x-a,t-b) also solution $\forall$ a,b $\in$ R

(ii) If U(x,t) is solution of wave equation then U(ax , bt) also solution if a = b

Qus. 47

Sol:- (c) For given matrix $A = \begin{pmatrix} 1 & 2 \\ 8 & 9 \end{pmatrix}$

$D=\begin{pmatrix}1&0\\0&9\end{pmatrix}$; $L=\begin{pmatrix}0&0\\8&0\end{pmatrix}$ & $U=\begin{pmatrix}0&2\\0&0\end{pmatrix}$ and it's Gauss-Seidel iteration matrix is $G=-(D+L)^{-1}U$

$$=-\begin{pmatrix}1&0\\8&9\end{pmatrix}^{-1}\begin{pmatrix}0&2\\0&0\end{pmatrix}$$

$$=-\frac{1}{9}\begin{pmatrix}9&0\\-8&1\end{pmatrix}\begin{pmatrix}0&2\\0&0\end{pmatrix}=\begin{pmatrix}0&-2\\0&16/9\end{pmatrix}$$

So, Eigen values of G are 0 & $\frac{16}{9}$

Qus. 48
Sol:- (c)

$$J(y)=\int_{-1}^{0}\left(12\,xy-\left(\frac{dy}{dx}\right)^2\right)dx$$

$y(0)=0 \qquad y(-1)=1$

Euler equation given by

$$\frac{\partial}{\partial y}f-\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y'}\right)=0 \qquad (1)$$

where $f=12xy-(y')^2$

$$\therefore \qquad \frac{\partial f}{\partial y}=12x-\frac{\partial}{\partial x}(-2y')=0$$

$\Rightarrow \quad 6x+\frac{\partial}{\partial x}y'=0$

$\Rightarrow \quad 6x+y''=0$

$\Rightarrow \quad y''=-6x$

$\Rightarrow \quad y'=-3x^2$

$\Rightarrow \quad y=-x^3$

Qus. 49
Sol:- (d)

Qus. 50
Sol:- (b)

Qus. 51
Sol:- (c)

Qus. 52
Sol:- (d)

Qus. 53
Sol:- (c)

Qus. 54
Sol:- (d)

Qus. 55
Sol:- (a) Primal Problem is

Maximize x_1+3x_2

Subject to $x_1+x_2\le 6$

$2x_1+x_2\le 8$

$x_1+2x_2\le 9$

$x_1\ge 0$ & $x_2\ge 0$

So, it's dual problem will be

Minimise: $W=6y_1+8y_2+9y_3$

Subject to $y_1+2y_2+y_3\ge 1$

$y_1+y_2+2y_3\ge 3$

$y_1,y_2,y_3\ge 0$

(option A)

Qus. 56
Sol:- (c)

Qus. 57
Sol:- (c)

Qus. 58
Sol:- (b)

Qus. 59
Sol:- (a)

$$P(\text{Head}\mid Y=15)=\frac{P((\text{Head})\cap(Y=15))}{P(Y=15)}$$

$$=\frac{\frac{1}{2}\times 100_{C_{15}}\times\left(\frac{1}{6}\right)^{15}\left(\frac{5}{6}\right)^{85}}{\frac{1}{2}\times 100_{C_{15}}\left(\frac{1}{6}\right)^{15}\left(\frac{5}{6}\right)^{85}+\frac{1}{2}\times 101C_{15}\left(\frac{1}{6}\right)^{15}\left(\frac{5}{6}\right)^{86}}$$

$$=\frac{100_{C_{15}}}{100_{C_{15}}+101_{C_{15}}\times\left(\frac{5}{6}\right)}=\frac{1}{1+\frac{101}{86}\times\frac{5}{6}}$$

$$=\frac{516}{516+505}=\frac{516}{1021}$$

Qus. 60
Sol:-(d)

PART "C"

Qus. 61
Sol:- (a) (b) (d)

1) $a_n = \frac{1}{n} \Rightarrow \limsup a_n = 0$

& $\liminf a_n = 0 \Rightarrow -\liminf a_n = 0$

So, $\limsup a_n = -\liminf a_n$

2) $a_n = (-1)^n \left(1 + \frac{1}{n}\right)$

$\Rightarrow \limsup a_n = 1$

& $\liminf a_n = -1$

$\Rightarrow \limsup a = -\liminf a_n$

3) $a_n = 1 + \frac{(-1)^n}{n}$

$\Rightarrow \limsup a_n = 1$

& $\liminf a_n = 1$

$\Rightarrow \limsup a_n \neq -\liminf a_n$

4) $a_n =$ enumeration of all rationals in $(-1,1)$

$\Rightarrow \limsup a_n = 1$ & $\liminf a_n = -1$

$\Rightarrow \limsup a_n = -\liminf a_n$

Qus. 62
Sol:- (a) (d)

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} (x-y)^2 \sin\left(\frac{1}{x-y}\right)$$

$= 0 = f(0,0)$

[$\because$ sin oscillates from -1 to 1]

so, $f(x,y)$ is continuous at $(0,0)$

$$f_x(0,0) = \lim_{h\to0} \frac{f(h,0)-f(0,0)}{h} =$$

$$\lim_{h\to0} h^2 \sin\frac{1}{h} = 0$$

$$f_y(0,0) = \lim_{K\to0} \frac{f(0,K)-f(0,0)}{K} =$$

$$\lim_{K\to0} K^2 \sin\left(\frac{1}{-K}\right) = 0$$

$$df = f(h,K) - f(0,0) = (h^2 - K^2)\sin\left(\frac{1}{h-K}\right) =$$

$$f_x(0,0).\, h + f_y(0,0).K + \sqrt{h^2+K^2}\; g(h,K)$$

$$\Rightarrow \quad g(h,K) = \frac{(h^2-K^2)\sin\left(\frac{1}{h-K}\right)}{\sqrt{h^2+K^2}}$$

$$\& \lim_{(h,K)\to(0,0)} g(h,K) = \lim_{r\to0^+} \frac{r^2(\cos^2\theta - \sin^2\theta)}{r}$$

$$\sin\frac{1}{r(\cos\theta - \sin\theta)} = 0$$

So, $f(x,y)$ is differentiable at $(0,0)$

Qus. 63
Sol:- (d)

$S1$: $e^{\cos t} = e^{2022 \sin t}$

$\Rightarrow \tan t = \frac{1}{2022} \Rightarrow \exists\, t \in (0,\pi)$

Satisfying it so $S1$ is wrong

$S2$: Take $f(x) = x - \log_e(1 + x e^t)$

I:- At $t = 0$; $f(x) = x - \log_e(1+x)$

$\Rightarrow f'(x) = 1 - \frac{1}{1+x} = \frac{x}{1+x} > 0$

So $f(x)$ is strictly increasing

so, $f(x) > f(0)$; $\forall x > 0$

$\Rightarrow x - \log_e(1+x) > 0 \Rightarrow x > \log_e(1+x)$

$\forall\; x > 0$ (1)

II:- At $t=x$; $f(x)=x-\log_e(1+xe^x)$

$\Rightarrow$ $f'(x)=1-\dfrac{e^x+xe^x}{1+xe^x}=\dfrac{-(e^x-1)}{1+xe^x}$

so, $f'(x)<0$; $\forall x>0$, so $f(x)$ is strictly decreasing $\forall\ x>0$.

So, $f(x)<f(0)$; $\forall x>0$

$\Rightarrow$ $x-\log_e(1+xe^x)<0$; $\forall x>0$

$\Rightarrow$ $x<\log_e(1+xe^x)$; $\forall\ x>0$ (2)

(1) & (2) $\Rightarrow$

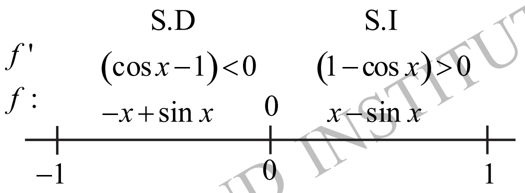
$\log_e(1+xe^0)<x<\log_e(1+xe^x)$

$\Rightarrow$ $\exists\ t\in(0,x)$ s.t.

$x=\log_e(1+xe^t)$

$S3$: e^x is monotonically increasing function

Let $f(x)=|x|-|\sin x|$ in $(-1,1)$



By the figure $f(x)\geq f(0)$; $\forall x\in(-1,1)$

$\Rightarrow$ $|x|-|\sin x|\geq 0$ $\Rightarrow$ $|x|\geq|\sin x|$

$\Rightarrow$ $e^{|x|}\geq e^{|\sin x|}$; $\forall x\in(-1,1)$

Qus. 64
Sol:- (a) (d)

$A=\left\{x\in[1,\infty)\ ;\ \lim_{n\to\infty}[x]^n \text{ exists}\right\}$

$=[1,2)$

$\Rightarrow$ $m(A)=1$ which is non-zero

$B=\left\{x\in[1,\infty)\ ;\ \lim_{n\to\infty}\left[x^n\right] \text{ exists}\right\}$

$=\{1\}$

& $m(B)=0$

$C=\left\{x\in[1,\infty)\ ;\ \lim_{n\to\infty}n[x]^n \text{ exists}\right\}$

$=\phi$

& $m(C)=0$

$D=\left\{x\in[1,\infty)\ ;\ \lim_{n\to\infty}[1-x]^n \text{ exists}\right\}$

$=[1,2]$

& $m(D)=1$ which is non zero

Qus. 65
Sol:- (a) (d)

$\Omega=\bigcup_{i=1}^{5}(i,i+1)=(1,2)\cup(2,3)\cup(3,4)\cup(4,5)\cup(5,6)$

So, Ω has 5 components, hence $1\leq|f(\Omega)|\leq 5$; $\because f'(x)=0$ $\forall\ x\in\Omega$

So, $(g\circ f)(\Omega)$ will contain at least 1 element and at most 5 elements.

So, $g\circ f(\Omega)$ is compact set (1)

and for set $5=\left\{e^x\ ;\ x\in(g\circ f)(\Omega)\right\}$

$1\leq|S|\leq 5$

So, it do not contain any nonempty interval.

Qus. 66
Sol:- (d)

$W=\{c_1\sin z+c_2\cos z+c_3\sin(2z)+c_4\cos(2z)\,|\ c_1,c_2,c_3,c_4\in c\}$

& Set $S=\{\sin z,\cos z,\sin(2z),\cos(2z)\}$ is linearly independent so

$\dim(W)=4$

Now matrix of linear operator $T:W\to W$ given by

$T(f(x))=f'(x)$ is

$$T = \begin{array}{c} \\ \sin z \\ \cos z \\ \sin 2z \\ \cos 2z \end{array} \begin{array}{cccc} T(\sin z) & T(\cos z) & T(\sin 2z) & T(\cos 2z) \\ \left[\begin{matrix} 0 \\ 1 \\ 0 \\ 0 \end{matrix}\right. & \begin{matrix} -1 \\ 0 \\ 0 \\ 0 \end{matrix} & \begin{matrix} 0 \\ 0 \\ 0 \\ 2 \end{matrix} & \left.\begin{matrix} 0 \\ 0 \\ -2 \\ 0 \end{matrix}\right] \end{array}$$

Eigen values of T are $i, -i, 2i$ & $-2i$

So, T is diagonalisable and Jordan Cahonical Form of T is

$$\begin{bmatrix} i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \\ 0 & 0 & 2i & 0 \\ 0 & 0 & 0 & -2i \end{bmatrix}$$

So it has 4 Jordan blocks

Qus. 67

Sol:- (a) (c) (d)

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{r\to 0} r^{2/3} (\cos\theta \sin\theta)^{1/3} = 0$$

$$= f(0,0)$$

So, $f(x,y)$ is continuous at $(0,0)$

(option B)

$$D_Q\left(f(0,0)\right) = \lim_{t\to 0^+} \frac{f(t\cos\theta, t\sin\theta) = f(0,0)}{t}$$

$$= \lim_{t\to 0^+} \frac{t^{2/3}(\cos\theta \sin\theta)^{1/3}}{t}$$

$$= \lim_{t\to 0^+} \frac{(\cos\theta \sin\theta)^{1/3}}{t^{1/3}} \quad \text{exist iff}$$

$$\cos\theta = 0 \text{ or } \sin\theta = 0 \Rightarrow \theta = 0, \frac{\pi}{2}, \pi \;\&\; \frac{3\pi}{2}$$

(option A)

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{0-0}{h} = 0$$

$$f_y(0,0) = \lim_{k\to 0} \frac{f(0,k) - f(0,0)}{k} = \lim_{k\to 0} \frac{0-0}{h} = 0$$

Now $df = f(h,k) - f(0,0) = h^{1/3} k^{1/3} - 0 =$

$$f_x(0,0).h + f_y(0,0).k + \sqrt{h^2+k^2}\; g(h,k)$$

$$\Rightarrow \quad g(h,k) = \frac{h^{1/3} k^{1/3}}{\sqrt{h^2+k^2}}$$

$$\lim_{(h,k)\to(0,0)} g(h,k) = \lim_{r\to 0^+} \frac{r^{2/3} \cos^{1/3}\theta \sin^{1/3}\theta}{r}$$

which does not exist ; $\forall \theta \in [0, 2\pi)$

So, $f(x,y)$ is not differentiable at $(0,0)$

Qus. 68

Sol:- (c)

Rank $\left(A^{n^2}\right) = 0 \qquad \Rightarrow A^{n^2} = 0$

$\Rightarrow$ A is nilpotent matrix

Now, A has n linearly independent eigen vectors where order $(A) = n$

then A is diagonalisable.

If A is diagonalisable nilpotent operator then A must be null matrix.

i.e. $A = O_n$

Qus. 69

Sol:- (c) Method I:

$$\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} \frac{1+2^\alpha+3^\alpha+\ldots+n^\alpha}{n^\alpha+1}$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \left(\frac{r}{n}\right)^\alpha$$

$$= \int_0^1 x^\alpha \, dx \quad \text{(By definite integral as limit of sum)}$$

$$= \frac{x^{\alpha+1}}{\alpha+1}\Bigg|_0^1 = \frac{1}{\alpha+1}$$

Method II:-

$$\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} \frac{1+2^\alpha+3^\alpha+\ldots+n^\alpha}{n^\alpha+1}$$

$$= \lim_{n\to\infty} \frac{(n+1)^\alpha}{(n+1)^{\alpha+1} - (n)^{\alpha+1}} = \frac{1}{\alpha+1}$$

(By Stolz- cesro theorem)

Qus. 70

Sol:- (d)

$\left\{1, x, \frac{1}{2!}x^2, ..., \frac{1}{n!}x^n\right\}$ is an orthonormal basis, so

$\left\langle x^i, x^j \right\rangle = 0$ whenever $i \neq j$

& $\left\langle \frac{x^i}{i!}, \frac{x^i}{i!} \right\rangle = 1$

$\Rightarrow \left(\frac{1}{i!}\right)^2 \left\langle x^i, x^i \right\rangle = 1$

$\Rightarrow \left\langle x^i, x^i \right\rangle = (i!)^2$

So, $\langle f, g \rangle = \left\langle \sum_{i=0}^{n} \alpha_i x^i, \sum_{i=0}^{n} \beta_i x^i \right\rangle =$

$\sum_{i=1}^{n} (i!)^2 \alpha_i \ \beta_i$

Qus. 71

Sol:- (a) (c) Standard basis of V is

$B_V = \{1, x, y, x^2, xy, y^2\}$

Matrix of linear transformation $T: V \to V$ is given by

$$T = \begin{array}{c} \\ 1 \\ x \\ y \\ x^2 \\ xy \\ y^2 \end{array} \begin{array}{cccccc} \frac{\partial 1}{\partial x} & \frac{\partial x}{\partial x} & \frac{\partial y}{\partial x} & \frac{\partial x^2}{\partial x} & \frac{\partial xy}{\partial x} & \frac{\partial y^2}{\partial x} \end{array} \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ Rank $(T) = 3$ & Nullity $(T) = 3$

Qus. 72

Sol:- (b) (c) (X, d) is a metric space and $|X| > 1$ & $|X|$ is finite.

Hence $X' = \phi$, so X is closed and bounded hence compact.

Also X is not connected.

Also, every function $f: X \to R$ is continuous.

Also every subset of X is open

Qus. 73

Sol:- (c)

$\frac{d}{dx}\cos x = -\sin x$ & $|-\sin x| \leq 1$

and $|-\sin x|^{\max} = 1$ is obtained at $x = \frac{\pi}{2}, \frac{3\pi}{2}, ...$ so largest change in $\cos x$ will be around $\frac{\pi}{2}$

i.e. in a nbd of $\frac{\pi}{2}$

If $x = \frac{\pi}{4}$ and $y = \frac{3\pi}{4}$ then

$|\cos x - \cos y| = \left|\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right| = \sqrt{2}$

& $|x - y| = \left|\frac{\pi}{4} - \frac{3\pi}{4}\right| = \frac{\pi}{2}$

So, $|\cos x - \cos y| < \sqrt{2}$ whenever $|x - y| < \frac{\pi}{2}$

Qus. 74

Sol:- (a) (b) Liner transformation

$T: R^n \to R^n$ is given by

$T(V) = AV \ ; \forall \ V \in R^n$

$\Rightarrow T^2(V) = A^2 V \ ; V \in R^n$

Now Rank $(A) =$ Rank(A^2)

$\Rightarrow$ Rank $(T) =$ Rank (T^2)

$\Rightarrow$ Nullity $(T) =$ Nullity (T^2) (option 2)

Also, Nullity $(T) \leq$ Nullity (T^2)

$\because \ N(T) \subseteq N(T^2)$

$\therefore \ N(T) = N(T^2)$ (option 1)

Qus. 75
Sol:- (a) (d)

If $x = \sum_{i=1}^{100} x_i e_i$ & $y = \sum_{i=1}^{100} y_i e_i$

then $B(x,y) = \sum_{i=1}^{100} x_i y_i$

and matrix of bilinear form is.

$$B = \begin{bmatrix} x_1y_1 & x_1y_2 & \cdots & x_1y_{100} \\ x_2y_1 & x_2y_2 & \cdots & x_2y_{100} \\ \vdots & & & \\ \vdots & & & \\ x_{100}y_1 & \cdots & \cdots & x_{100}y_{100} \end{bmatrix}$$

$$\& \ B(x,x) = \begin{bmatrix} x_1^2 & \cdots & \cdots & x_1x_{100} \\ & x_2^2 & & \\ & & \ddots & \\ x_{100}\,x_1 & & & x_{100}^2 \end{bmatrix}$$

So, $B(x,x) \neq 0 \ ; \forall x \in c^{100}$

so, B is non degenerate

& Rank $(B) = 1$

Now, $A \neq 0$ & $A^2 = 0 \Rightarrow$

Rank$(A^2) \geq 2$ Rank$(A)-$ order(A)

$\Rightarrow$ $0 \geq 2$ Rank $(A) - 100$

$\Rightarrow$ Rank $(A) \leq 50$ (option 4 is correct)

(option 3 is incorrect)

$\because$ option (4) is correct so option (2) is incorrect

Qus. 76
Sol:- (a) (c) (d)

$\because$ $\dim(U) = \dim(V) = 2$

$\therefore$ If $B_U = \{\alpha_2, \alpha_2\}$ & $B_V = \{\beta_1, \beta_2\}$

be bases of $U(R)$ & $V(R)$ then

Linear transformation $T : R^3 \to R^3$

given by $T(\alpha_1) = \beta_1$

& $T(\alpha_2) = \beta_2$, & $T(\alpha_3) = \beta_3$

where $\{\alpha_1, \alpha_2, \alpha_3\}$ & $\{\beta_1, \beta_2, \beta_3\}$ are bases of R^3, is invertible linear transformation such that

$T(U) = V$

So, option (1) is correct but option (2) is incorrect.

$\exists\, a$ linear transformation $T : R^3 \to R^3$ s.t.

$T(U) \cap V \neq \{0\}$ and characteristic polynomical of T has 2 non real roots so it is not product of linear polynomials with real coefficients.

Characteristic polynomial of T including factor $(x-1)$ exist satisfying

$T(U) = V$.

Qus. 77
Sol:- (b) (d)

Finite union of proper subspaces will be a vector space then it will be also one of the proper subspaces of $M_n(R)$, so it cannot be equal to $M_n(R)$

option 1 is false

$\det(Y I + A)$ will be equal to zero for at most n values of y, so $\forall x \in R$ & $\in > 0$ $\exists y \in (x - \in,\ x + \in)$ such that

$\det(Y I + A) \neq 0$

If W is a subspace of $M_n(R)$ then there exist a linear transformation

$T : M_n(R) \to M_n(R)$

such that Range $(T) = W$.

Qus. 78
Sol:- (a) (c) (d)

If f is uniformly continuous in (a,b) then either f is constant

Or $\exists \alpha > 0$ s.t.

$|f(x) - f(y)| \leq \alpha |x - y|$

So, $\exists\ \alpha \geq 0$ & $\beta \geq 0$ such that

$|f(x) - f(y)| \leq \alpha\ |x - y| + \beta$ where

$\{\alpha_1,\alpha_2,\alpha_3\}$ & $\{\beta_1,\beta_2,\beta_3\}$ are bases of R^3, is invertible linear transformation such that

(option 1 is correct)

$f(x)=\frac{1}{x}$ is continuous in $(0,1)$ and if $[c,d]\subseteq(0,1)$ then $f(x)$ is uniformly continuous in $[c,d]$ but not uniformly continuous in $(0,1)$

(option 2 is incorrect)

If f is strictly increasing and bounded then $\lim_{x\to a^+} f(x)$ and $\lim_{x\to b^-} f(x)$ exist, so it is uniformly continuous in (a,b)

(option 3 is correct)

If f is uniformly continuous in (a,b) then f maps every Cauchy sequence into Cauchy sequence in R so it will be convergent sequence

(option 4 is correct)

Qus. 79

Sol:- (a) (d)

$a>b$ & $a+b=24$

$2x\equiv 3a(\text{mod}\,5) \Rightarrow 3.2x\equiv 3.3a(\text{mod}\,5)$

$\Rightarrow$ $x\equiv 4a(\text{mod}\,5) \Rightarrow x=4a+t_1 5;\ t_1\in I$

& $x\equiv 4b(\text{mod}\,5) \Rightarrow x=4b+t_2 5\ ;\ t_2\in I$

$\Rightarrow$ $4(a-b)=(t_2-t_1)5$

$\Rightarrow$ $(a-b)$ is multiple of 5

$\Rightarrow$ $a=b+5t\ ;\ t=1,2,3,4$

$\left.\begin{matrix}a-b=5\\ a+b=24\end{matrix}\right\}$ has no integral solution

$\left.\begin{matrix}a-b=10\\ a+b=24\end{matrix}\right\}\Rightarrow a=17$ & $b=7$

$\left.\begin{matrix}a-b=15\\ a+b=24\end{matrix}\right\}$ has no integral solution

$\left.\begin{matrix}a-b=20\\ a+b=24\end{matrix}\right\}\Rightarrow a=22$ & $b=2$

So, $10\le a-b\le 20$

Qus. 80

Sol:- (a) (c)

If $z_0\in\Omega$ then $f(z)$ will be identically 0 in Ω. So, if $f(z)$ is not identically zero in Ω then $z_0\in\partial\Omega$

If $\lim f(z_k)=f(z_0)=0$ i.e. if set of zeroes of $f(z)$ which are forming a sequence converges to a find inside the domain then $f(z)=0$ identically in Ω. So, there exist $r>0$ such that on $|z-z_0|=r\ ;\ f(z)=0$

Qus. 81

Sol:- (a) (b) (c)

$$A=\frac{\mathbb{Z}[x]}{\langle x^2+x+1,x^3+2x^2+2x+6\rangle}$$

$x^2+x+1\mid x^3+2x^2+2x+6$

$\Rightarrow$ $(x^3+2x^2+2x+6)-(x^2+x+1)(x+1)-5=0$

$\Rightarrow$ $(x^3+2x^2+2x+6)-(x^2+x+1)(x-1)=5$

$\Rightarrow$ $\langle x^2+x+1,x^3+2x^2+2x+6\rangle=\langle x^2+x+1,5\rangle$

$\therefore$ $$\frac{\mathbb{Z}[x]}{\langle x^2+x+1,5\rangle}\approx\frac{\mathbb{Z}_5[x]}{\langle x^2+x+1\rangle}$$

$\Rightarrow$ x^2+x+1 is irreducible over $\mathbb{Z}_5$

$\therefore$ A is field

Thus (a) (b) (c) are correct option

Qus. 82

Sol:- (a) (b) (c)

Ω is open subset of C containing 0 (zero)

By the concept of open mapping theorem $\{f(z)=e^z\ ;\ z\in\Omega\}$ is an open subset and also $\{f(z)=\sin z\ ;\ z\in\Omega\}$ is an open subset of C, because e^z and $\sin z$ both are analytic function in Ω.

Also there is no zero of e^z so $f(z)=e^z$ satisfy maximum and minimum modulus prin cipal so $\left|e^z\right|$ has both maxima and minima or boundary of Ω. Thus set $S=\left\{\left|e^z\right| ; z\in\Omega\right\}$ is open subset of R.

But if $\Omega=\{z\in C \mid |z|<10\}$ then set $T=\{|\sin z| ; z\in\Omega\}=[-1,1]$ which is not open subset of R.

Qus. 83

Sol:- (a) (b) (d)

$$f(n)=n^5-2n^3+n=n\left(n^4-2n^2+1\right)$$

$$=n\left(n^2-1\right)^2=n(n-1)^2(n+1)^2$$

For every $K\in N$, $\exists\ n=2^K\in N$ such that 2^K divides

$$f(n)=f\left(2^K\right)=2^K\left(2^K-1\right)^2\left(2^K+1\right)^2$$

(option a)

$\forall\ n\geq 20$; if n is odd then $n-1$ & $n+1$ are even and if n is even that n is even, so $f(n)$ is always even.

(option b)

But if $n=26$ then $f(n)=26(25)^2(27)^2$ which is neither odd nor multiple of 4 so option (c) is incorrect

If n is odd integer ≥ 21 then if $n=2m+1$ then $n-1=2m$ and

$n+1=2m+2=2(m+1)$, so

$$f(n)=f(2m+1)=(2m+1)(2m)^2(2m+2)^2$$

$$=(2m+1)(16)(m(m+1))^2$$

(As $m(m+1)$ is divisible by $2!$) so $f(n)$ is divisible by $(16)(2!)^2=64$

(option d)

Qus. 84

Sol:- (a) (b)

$$O(G)=2022=2\times3\times337$$

$$n_2=1+2K/1011\Rightarrow n_2=1,3$$

$$n_3=1+3K/674\Rightarrow n_3=1$$

$$n_{337}=1+337K/6\Rightarrow n_{337}=1$$

Let $a\in G$ and $O(a)=odd$ then s_g is an even permutation is correct.

$H=\{g\in G \mid O(g)=odd\}$ is noraml subgroup of G is correct.

The subgroup of order 337 is normal but not index 337.

G has more than 2 normal subgroups.

Qus. 85

Sol:- (a) (b) (c) (d)

$$\left(f^n\right)'=f'\left(f^{n-1}\right).f'\left(f^{n-2}\right).f'\left(f^{n-3}\right)\ldots$$

$f'(f), f'$ so

$$\left(f^n\right)'(0)=f'\left(f^{n-1}(0)\right)f'\left(f^{n-2}(0)\right),$$

$$f'\left(f^{n-3}(0)\right)\ldots.f'\left(f(0)\right)\ldots f'(0)$$

$$=\underbrace{f'(0).f'(0).f'(0)\ldots f'(0)}_{n\ times}=\left(f'(0)\right)^n$$

$\because\quad f(0)=0$ (option a)

$$\because\quad \left|f'(z)\right|\leq\frac{1-|f(z)|^2}{1-|z|^2}$$

$$\therefore\quad \left|f'(0)\right|\leq\frac{1-|f(0)|^2}{1-|0|^2}$$

$\Rightarrow\quad |f'(0)|\leq 1$ (option d)

$\because\quad |f'(0)|\leq 1\ \Rightarrow\ |f'(0)|^n\leq 1$

$\therefore$ The sequence $\left(\left(f'(0)\right)^n\right)_n$ is bounded

(option b)

$\because$ Codomain of f is U so $f^n(U)\subseteq U$ but is $f(z)=z$ then $f^n(U)=U$. so option b will be correct only if C stands for subset & not for proper subset contradiction

$f(z)=z\Rightarrow f\circ f\circ\ldots f(z)=z$ so in this case $f(U)=U$.

Qus. 86
Sol:- (a) (c)

$f(z)$ is entire function satisfying

$(f(z))^2 + (f'(z))^2 = 1$ and

$X = \{z : f'(z) = 0\}$

$Y = \{z : f''(z) + f(z) = 0\}$

Now $(f'(z))^2 = 1 - (f(z))^2$

$\Rightarrow \dfrac{f'(z)}{\sqrt{1-(f(z))^2}} = 1$ if $f(z) \neq \pm 1$

$\Rightarrow \int \dfrac{f'(z)}{\sqrt{1-(f(z))^2}}\, dz = \int 1\, dz + c$

$\Rightarrow \sin^{-1}(f(z)) = z + c$

$\Rightarrow f(z) = \sin(z+c)$ or $f(z) = 1$ or $f(z) = -1$

where c is any complex constant

(Case I)

If $f(z) = \sin(z+c)$ then

$X = \{z \in c \mid \cos(z+c) = 0\}$ has no limit point

& $Y = \{z \in c \mid f''(z) + f(z) = 0\} = c$

whose derived set is c

(Case II)

If $f(z) = I\, 1$ then

$X = \{z \in c \mid 0 = 0\} = c$ whose derived set is c

& $Y = \phi$ $\Rightarrow Y' = \phi$.

Qus. 87
Sol:- (b) (c)

$\overline{\mathbb{F}_p} \rightarrow$ algebraic closure field

$\overline{\mathbb{F}_p} \bigcup_{r=1}^{\infty} \mathbb{F}_{p^r}$

$\delta = \{\mathbb{F} \subseteq \overline{\mathbb{F}_p} \mid [\mathbb{F} : \mathbb{F}_p] < \infty\}$

$[\mathbb{F} : \mathbb{F}_p] = n \quad |\mathbb{F}| = p^n$

$\therefore \quad \delta = \{\mathbb{F}_p, \mathbb{F}_{p^2}, \ldots\}$

$\therefore \quad \delta$ is countable

$\therefore$ (b) (a) discard

subfield of $\mathbb{F}_{p^r}$ is inform $\mathbb{F}_{p^k}$ where K/r

Let $\mathbb{F}_{p^2}$ and $\mathbb{F}_{p^3}$

$\mathbb{F}_{p^2} \not\subseteq \mathbb{F}_{p^3} \qquad 2\chi 3$

$\therefore$ (c) discard

If $n > 1\ \exists\, F \in \sigma$ such that $[\mathbb{F} : \mathbb{F}_p] = n$

$\mathbb{F} = \mathbb{F}_{p^n}$ and $[\mathbb{F}_{p^n} : \mathbb{F}] = n$

$\therefore$ (d)

Qus.88
Sol:-(a) (c)

$2 \nmid 15$ so option (d) discard

$\circ(G) = 14$ *and* $\circ(Z(G)) = 2$

$\left|\dfrac{G}{Z(G)}\right| \approx Z_7$ thus abelian

so option (b) discard

Result :- $\circ(G) = pq \quad p \mid q$ then class equation

$1 + \underbrace{p + p + \ldots + p}_{\frac{q-1}{2}\ times} + \underbrace{q + q + \ldots + q}_{remais}$

thus option (a) and (c) correct

Qus. 89
Sol:- (a) (d)

If X is finite set then each point of X is an isolated point so every function $f : X \rightarrow R$ is continuous.

(option a is correct)

If $X = R$ then

$f : X \rightarrow R$ given by $f(x) = x$ is continuous but X is not finite

(option b is incorrect)

Not $x = (x_1, x_2, x_3, x_4, x_5)$

$X = \left\{(x_1, 0, 0, 0, 0) \mid x_1 \in \left\{\dfrac{1}{n} \mid n \in N\right\} \cup \{0\}\right\}$

is compact set and infinite set but it is countable

(option c is incorrect)

If $\left|X\right| \geq 2$ and X is connected then X must be uncountable points.

(option d is correct)

Qus. 90
Sol:- (a) (b) (c)

Qus. 91
Sol:- (a) (d)

Qus. 92
Sol:- (a) (c)

$$x = \{u \in c'[0,1]\,;\, u(0) = u(1) = 0\}$$

$I : X \to \mathbb{R}$ defined as

$$I(u) = \int_0^1 e^{-u;(t)^2}\, dt \;\forall\; u \in X$$

we know that if $f(t,u,u')$ is independent of x and y i.e. depand only on u' then extremal is given by

$u = a\,t + b$ (1)

Given that $f(t,u,u') = e^{-u'^2}$

from (1) $u(0) = 0$, $u(1) = 0$

$\therefore$ $a = 0$ $b = 0$

$\therefore$ $u = 0$ $u' = 0$

$\therefore$ $I(u) = \int_0^1 e^0\, dt = \int_0^1 1\,dt = 1$

Now check weather 1 is maxima or minima

$$f = e^{-u'^2}$$

$$\frac{\partial f}{\partial u'} = e^{-u'^2}(-2u')$$

$$\frac{\partial^2 f}{\partial u'^2} = -2\left[e^{-u'^2}(-2u')u' + e^{-u'^2}\right]$$

$$= -2\left[e^0(-2,0)\,0 + e^0\right] = -2 < 0$$

$\therefore$ Maximum $M = 1$
Now for minimum value
& we know that
exponantial always greater than 0

$\therefore$ $m > 0$

$\therefore$ $I(u) \in (0,1)$

(a) is correct
M is attained but not m.
(c) is correct

Qus. 93
Sol:- (a) (d)

$$u_x + x\,u_y = 0$$

$$u(x,0) = e^x$$

$P = 1$ $Q = x$ $R = 0$

$x = t$ $y = 0$ $u = e^t$

$$\frac{dx}{1} = \frac{dy}{x} = \frac{du}{0}$$

$$\frac{dx}{1} = \frac{dy}{x} \Rightarrow \frac{x^2}{2} - y = c_1$$

$$\frac{dx}{1} = \frac{du}{0} \Rightarrow u = c_2$$

$\therefore$ $c_1 = \dfrac{t^2}{2}$ & $c_2 = e^t$

$\Rightarrow$ $t = \pm\sqrt{2c_1}$ $\therefore c_2 = e^{\pm\sqrt{2c_1}}$

$\Rightarrow$ $\therefore u = e^{\pm\sqrt{2\left(\frac{x^2}{2} - y\right)}}$

(i) $u(2,1) = e^{\pm\sqrt{2(2-1)}} = e^{\pm\sqrt{2}}$

(ii) $u\left(1,\dfrac{1}{2}\right) = e^{\pm\sqrt{2\left(\frac{1}{2} - \frac{1}{2}\right)}} = e^0 = 1$

(iii) $u(-2,1) = e^{\pm\sqrt{2(2-1)}} = e^{\pm\sqrt{2}}$

Qus. 94
Sol:- (d)

$y(t)$ is satationary function

$$J[y] = \int_{-1}^1 (1 - x^2)(y')^2\, dx$$

$y(-1) = 1$, $y(1) = 1$ subject to $\int_{-1}^1 y^2 = 1$

(b) Let $y = x^2$ then

$y(-1) = 1$, $y(1) = 1$

Now

$$\int_{-1}^1 y^2 = \int_{-1}^1 x^4\, dx = \left(\frac{x^5}{5}\right)_{-1}^1 = \frac{1}{5}(1+1) = \frac{2}{5} \neq 1$$

$\therefore$ (b) discard

(c) Let $y = x$ then

$y(-1) = -1 \neq 1$

Not satisfying the boundary condition

$\therefore$ (c) discard

(a) If y is unique then it is either a odd order polynomial or even order polynomial.

but from above polynomial $y = x^2$ and $y = x$

we discard option (a)

Thus no such y exist

$\therefore$ (d) is correct

Qus. 95

Sol:- (a) (d)

$$F_{n-1} + F_{n-2} = \frac{\alpha^{n-1} - \beta^{n-1}}{\alpha - \beta} + \frac{\alpha^{n-2} - \beta^{n-2}}{\alpha - \beta}$$

$$\frac{\alpha^{n-2}(\alpha+1) - \beta^{n-2}(\beta+1)}{\alpha - \beta} = \frac{\alpha^n - \beta^n}{\alpha - \beta}$$

$= F_n$ (option a is correct)

$\because \quad \alpha = \frac{1+\sqrt{5}}{2} \Rightarrow \alpha + 1 = \frac{3+\sqrt{5}}{2} = \frac{6+2\sqrt{5}}{4}$

$= \alpha^2$

& $\beta = \frac{1-\sqrt{5}}{2} \Rightarrow \beta + 1 = \frac{3-\sqrt{5}}{2} = \frac{6-2\sqrt{5}}{4}$

$= \beta^2$

Now $p(1) = F_1 = 1$

$p(3) = F_3 = 1+1 = 2$

$p(5) = F_5 = 3+2 = 5$

x	y	Δ	Δ^2
1	1		
		1	
3	2		2
		3	
5	5		

$$\Rightarrow \quad p(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0$$

where $x = 1 + 2p \Rightarrow p = \frac{x-1}{2}$

$$\Rightarrow \quad p(x) = 1 + \frac{(x-1)}{2} + \frac{\left(\frac{x-1}{2}\right)\left(\frac{x-3}{2}\right)}{2} \times 2$$

$$\Rightarrow \quad p(7) = 1 + 3 + \frac{3\times 2}{2} \times 2 = 10$$

(option d is correct)

Qus. 96

Sol:- (a) (c)

Qus. 97

Sol:- (a) (c)

In RK method of order 2

$W_1 + W_2 = 1$

$$\left.\begin{aligned} W_2\,\alpha &= \frac{1}{2} \\ W_2\,\beta &= \frac{1}{2} \end{aligned}\right\} \Rightarrow \alpha = \beta$$

In option (a) & (c) all conditions are satisfied but they are not satisfied in option & (b) & (d)

Qus. 98

Sol:- (c)

g is solution of volterra integral equation

$$g(s) = 1 + \int_0^s (s-t)\,g(t)\,dt$$

Applying Laplace convolution theorem.

$L(f * g) = L(f) * L(g)$

$L(g(s)) = L(1) + L(t)\, L(g(s))$

$L(g(s)) = \frac{1}{s} + \frac{1}{s^2}\, L(g(s))$

$\Rightarrow \quad L(g(s))\left(1 - \frac{1}{s^2}\right) = \frac{1}{s}$

$\Rightarrow \quad L(g(s))\left(\frac{s^2-1}{s^2}\right) = \frac{1}{s}$

$\Rightarrow \quad L(g(s))\left(\frac{s^2-1}{s}\right) = 1$

Taking Laplce inverse both side

$\therefore \quad g(s) = L^{-1}\left(\frac{s}{s^2-1}\right) = \cosh t$

$$g(1)=\cos h1$$

$$=\frac{e+e^{-1}}{2}=\frac{1}{2}\left(e+\frac{1}{e}\right)$$

Qus. 99
Sol:- (a)

$$A=\begin{pmatrix}1 & 1\\ 4 & -2\end{pmatrix} \qquad h=\begin{pmatrix}3t+1\\ 2t+5\end{pmatrix}$$

$$\Rightarrow \quad y'=\begin{pmatrix}1 & 1\\ 4 & -2\end{pmatrix}\begin{pmatrix}y_1\\ y_2\end{pmatrix}+\begin{pmatrix}3t+1\\ 2t+5\end{pmatrix}$$

$$\Rightarrow \quad \frac{dy_1}{dt}=y_1+y_2+3t+1$$

$$\frac{dy_2}{dt}=4y_1-2y_2+2t+5$$

$$\Rightarrow \quad (D-1)y_1-y_2=3t+1 \qquad \text{(i)}$$

$$(D+2)y_2-4y_1=2t+5 \qquad \text{(ii)}$$

$$(1)\times 4 \; +(2)\times(D-1)$$

$$\Rightarrow \quad \left(D^2+D-6\right)y_2=10t+1$$

$$\Rightarrow \quad (D-2)(D+3)y_2=10t+1$$

$$\therefore \quad y_2(t)=c_1e^{2t}+c_2e^{-3t}+\text{P.I}$$

$$\Rightarrow \quad \frac{1}{D^2+D-6}(10t+1)$$

$$\Rightarrow \quad -\frac{1}{6}\left[1-\left(\frac{D^2+D}{6}\right)\right]^{-1}\times(10t+1)$$

$$\Rightarrow \quad -\frac{1}{6}\left[1+\frac{D^2+D}{6}+\ldots\right](10t+1)$$

$$\Rightarrow \quad -\frac{1}{6}\left[1+10t+\frac{1}{6}(10)\right]$$

$$=\frac{-1}{6}-\frac{5}{3}t-\frac{5}{18}=\frac{-5}{3}t-\frac{4}{9}$$

$$\therefore \quad y_2(t)=c_1e^{2t}+c_2e^{-3t}-\frac{5}{3}t-\frac{4}{9}$$

$$y_2'(t)=2c_1e^{2t}-3c_2e^{-3t}-\frac{5}{3}$$

$$2c_1e^{2t}-3c_2e^{-3t}-\frac{5}{3}=4y_1-2c_1e^{2t}-2c_2e^{-3t}$$

$$+\frac{10}{3}t+\frac{8}{9}+2t+5$$

$$\Rightarrow \quad y_1=\frac{1}{4}\left(4c_1e^{2t}-c_2e^{-3t}-\frac{16}{3}t-\frac{68}{9}\right)$$

$$\begin{pmatrix}y_1(t)\\ y_2(t)\end{pmatrix}=\begin{pmatrix}c_1e^{2t}-\frac{1}{4}c_2e^{-3t}-\frac{4}{3}t-\frac{17}{9}\\ c_1e^{2t}+c_2e^{-3t}-\frac{5}{3}t-\frac{4}{9}\end{pmatrix}$$

$$\lim_{t\to\infty}\frac{y(t)}{t}=\lim_{t\to\infty}\begin{pmatrix}c_1e^{2t}-\frac{1}{4}c_2e^{-3t}-\frac{4}{3}t-\frac{17}{9}\\ c_1e^{2t}+c_2e^{-3t}-\frac{5}{3}t-\frac{4}{9}\end{pmatrix}$$

$$=\begin{pmatrix}-\frac{4}{3}\\ -\frac{5}{3}\end{pmatrix}$$

Qus. 100
Sol:- (a) (b) (c)

Qus. 101
Sol:- (b) (c)

$\frac{dy}{dx}=y^{\alpha}\;\; y(0)=0 \quad 0<\alpha<1$ has infinite solution

$\frac{dy}{dx}=y^{\alpha}\;\; y(0)=a\neq 0 \quad 0<\alpha<1$ has unique solution

$\frac{dy}{dx}=y^{\alpha}\;\; y(0)=b \;\; \alpha\geq 1$ unique solution

$\frac{dy}{dx}=y^{\alpha}\;\; y(0)=0 \quad 0<\alpha<1$ infinite solution

$\therefore$ (i) has infinite solution

$y'(x)=y^{1/3} \quad y(0)=0$ infinite solution

(ii) $\frac{dy}{dx}=-y^{1/3} \quad y(0)=0$

$$\Rightarrow \quad \int y^{-1/3}\,dy=\int -dx$$

$\Rightarrow \quad \frac{3}{2}\, y^{2/3} = -x + c$

$\Rightarrow \quad \frac{3}{2} y^{2/3} + x = c$

Qus. 102
Sol;- (a) (b)

$\phi_1(x) = \sin x + \int_0^x \phi_2(t)\,dt$

$\phi_2(x) = 1 - \cos x - \int_0^x \phi_1(t)\,dt$

Now

$\phi_1(x) = \sin x + \int_0^x \left[1 - \cos t - \int_0^x \phi_1(t)\,dt\right] dt$

$= \sin x + \int_0^x dt - \int_0^x \cos t\,dt \;\; - \int_0^x \int_0^x \phi_1(t)\,dt$

$= \sin x + x - (\sin t)_0^x - \frac{1}{(2-1)!} \int_0^x (x-t)\,dt$

(by Leibnitz rule)

$= \sin x + x - \sin x - \int_0^x (x-t)\,dt$

$\phi_1(x) = x - \int_0^x (x-t)\,dt$

Applying Laplace convolution theorem

$L(f * g) = L(f) * L(g)$

$\therefore \quad L(\phi_1(x)) = L(x) - L(\int_0^x (x-t)dt$

$\overline{\phi_1}(x) = \frac{1}{s^2} - \frac{1}{s^2}\overline{\phi_1}(x)$

$\Rightarrow \quad \overline{\phi_1}(x) = \left(1 + \frac{1}{s^2}\right) = \frac{1}{s^2}$

$\Rightarrow \quad \overline{\phi_1}(x) = \left(\frac{s^2+1}{s^2}\right) = \frac{1}{s^2}$

$\Rightarrow \quad \overline{\phi_1}(x) = \frac{1}{s^2+1}$

Taking Lplace inverse both side

$\Rightarrow \quad \phi_1(x) = \sin x$

$\phi_1(x) = 0 \quad \Rightarrow \sin x = 0$

& $\mathbb{Z}$ is countable set

$\therefore$ ϕ_1 vanishes at most countably many point

$\therefore$ (a) (b) discard

$\phi_2(x) = 1 - \cos x - \int_0^x \phi_1(t)\,dt$

$= 1 - \cos x - \int_0^x \sin t\,dt$

$= 1 - \cos x + (\cos t)_0^x$

$= 1 - \cos x + \cos x - 1$

$= 0$

$\therefore \quad \phi_2(t) = 0$

$\therefore$ Vanishes at uncountable point

$\therefore$ (c) wrong

Qus. 103
Sol:- (a) (b)

Qus. 104
Sol:- (a) (c)

Qus. 105
Sol:- (a) (b) (c) (d)

Qus. 106
Sol:- (a) (c)

Qus. 107
Sol:- (a) (b) (c)

Qus. 108
Sol:- (a) (d)

Qus.. 109
Sol:- (a) (b) (d)

Qus. 110
Sol:- (a) (d)

Qus. 111
Sol:- (a) (b) (c) (d)

Qus. 112
Sol:- (c) (d)

Qus. 113
Sol:- (b) (d)

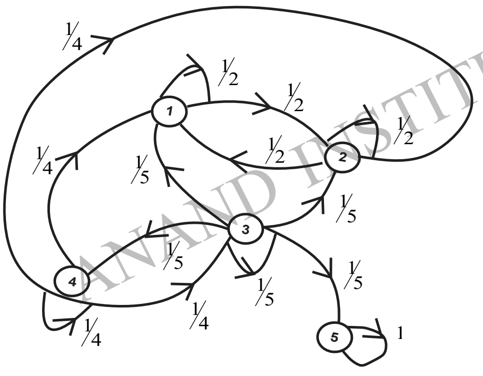
Qus. 114
Sol:- (a) (d)

Qus. 115
Sol:- (b) (c) (d)

Qus. 116
Sol:- (a) (c)

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

Transition diagram is



Recurrent $\to$ Returning to that state is possible

Transiant $\to$ Returing to state i is incontion

d) All state are recurrent (X)

$4 \to 2 \to 1 \not\to 4$ (can not return to 4 from state 1)

b) $1 \to 2 \to 1$, $3 \to 4 \to 3$, $5 \to 5$ $2 \to 1 \to 2$

$4 \to 3 \to 4$

$\therefore$ (b) discard

Recurrent $\{1,2,5\}$

Transiant $\{4,3\}$

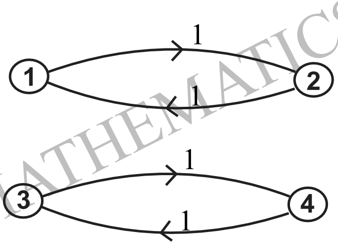
Qus. 117
Sol:- (a) (c) (d)

Qus. 118
Sol:- (a) (d)

Qus. 119
Sol:- (b) (d)

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

Transition diagram



(a) Irreduible Markov Chain
If all state are communicate with each other then irreducible other wise reduible.
State (1) & (3) do not communicate so it is reduible markov chain
(option a is wrong)

(b) Returing to state is possible so all state are recurrent.
(option b is right)

(c) $\lim\limits_{n\to\infty} p_{ij}^{(n)} = \pi_j$

$$\lim_{n\to\infty} p_{ij}^{n} = \begin{cases} 1 \ ; & n \ even \\ 0 \ ; & n \ odd \end{cases}$$

Limit does not exist.

(d) Clearly from transiant diagram all state have some period.

Qus. 120
Sol:- (a) (b)